

$$2.1 \quad p^\mu = (E, E \sin \theta \cos \phi, E \sin \theta \sin \phi, E \cos \theta)$$

$$p_{ab} = \begin{pmatrix} -p^0 + p^3 & p^1 - i p^2 \\ p^1 + i p^2 & -p^0 - p^3 \end{pmatrix} = \begin{pmatrix} -E + E \cos \theta & E \sin \theta \cos \phi - i E \sin \theta \sin \phi \\ E \sin \theta \cos \phi + i E \sin \theta \sin \phi & -E - E \cos \theta \end{pmatrix}$$

$$= E \begin{pmatrix} \cos \theta - 1 & \sin \theta \cos \phi - i \sin \theta \sin \phi \\ \sin \theta \cos \phi + i \sin \theta \sin \phi & -1 - \cos \theta \end{pmatrix}$$

$$= E \begin{pmatrix} \cos \theta - 1 & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -1 - \cos \theta \end{pmatrix}$$

$$(e^{i\theta})^2 = e^{2i\theta} = \cos 2\theta + i \sin 2\theta$$

$$\begin{aligned} \cos \theta - 1 &= \left(\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \right) - 1 \\ &= \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} - \left(\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \right) \\ &= -2 \sin^2 \frac{\theta}{2} \end{aligned}$$

$$= E \begin{pmatrix} -2 \sin^2 \frac{\theta}{2} & 2 \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ 2 \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & -2 \cos^2 \frac{\theta}{2} \end{pmatrix}$$

$$= 2E \begin{pmatrix} -\sin^2 \frac{\theta}{2} & \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & -\cos^2 \frac{\theta}{2} \end{pmatrix}$$

Maxwell's eqn: $p_{ab} p^b = 0$

$$\begin{pmatrix} -\sin^2 \frac{\theta}{2} & \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & -\cos^2 \frac{\theta}{2} \end{pmatrix} \cdot \sqrt{2} E \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix} = \begin{pmatrix} -\sin^2 \frac{\theta}{2} \cos \frac{\theta}{2} + \cos \frac{\theta}{2} \sin^2 \frac{\theta}{2} \\ \cos^2 \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} - \cos^2 \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$p^{ab} = p_{\mu} (\bar{\sigma}^{\mu})^{ab}$$

$$= \begin{pmatrix} p_0 - p_3 & -p_1 + ip_2 \\ -p_1 - ip_2 & p_0 + p_3 \end{pmatrix}$$

~~$$= \begin{pmatrix} p_0 - p_3 & -p_1 + ip_2 \\ -p_1 - ip_2 & p_0 + p_3 \end{pmatrix}$$~~

$$= \begin{pmatrix} -p^0 - p^3 & -p^1 + ip^2 \\ -p^1 - ip^2 & -p^0 + p^3 \end{pmatrix}$$

$$= \begin{pmatrix} -E - E\omega\theta & -E\sin\theta\omega\phi + iE\sin\theta\sin\phi \\ -E\sin\theta\omega\phi - iE\sin\theta\sin\phi & -E + E\omega\theta \end{pmatrix}$$

$$= E \begin{pmatrix} -1 - \omega\theta & \sin\theta(-\omega\phi + i\sin\phi) \\ \sin\theta(-\omega\phi - i\sin\phi) & -1 + \omega\theta \end{pmatrix}$$

$$= E \begin{pmatrix} -2\omega^2 \frac{\theta}{2} & -\sin\theta e^{-i\phi} \\ -\sin\theta e^{i\phi} & -2\sin^2 \frac{\theta}{2} \end{pmatrix}$$

$$= 2E \begin{pmatrix} -\omega^2 \frac{\theta}{2} & -\omega \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ -\omega \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & -\sin^2 \frac{\theta}{2} \end{pmatrix}$$

$$= -2E \begin{pmatrix} \omega^2 \frac{\theta}{2} & \omega \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ \omega \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & \sin^2 \frac{\theta}{2} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \omega \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & \sin^2 \frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} \frac{\theta}{2} \sin \frac{\theta}{2} & \sin^2 \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \omega e^{i\phi} & 1 \end{pmatrix}$$

$$\bar{\sigma}^{\mu} = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right)$$

$$= \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \right)$$

$$\eta^{\mu\nu} p_{\mu} = p^{\nu}$$

$$\eta_{\mu\nu} p^{\mu} = p_{\nu}$$

$$\eta_{\mu 3} p^{\mu} = p_3$$

$$\eta_{33} p^3 = p_3$$

$$1 \cdot p^3 = p_3$$

$$|p\rangle^a = (|p\rangle^i)^a = \sqrt{2}E \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{-i\phi} \end{pmatrix}$$

$$|p\rangle^a = \varepsilon^{ab} \langle p|_b$$

$$\varepsilon^{ca} |p\rangle^a = \varepsilon^{ca} \varepsilon^{ab} \langle p|_b$$

$$= \delta^b_c \langle p|_b = \langle p|_c$$

$$\Rightarrow \langle p|_b = \varepsilon_{ba} |p\rangle^a$$

$$\text{or } \langle p|_a = \varepsilon_{ab} |p\rangle^b$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{so } \langle p|_a = \sqrt{2}E \begin{pmatrix} -\sin \frac{\theta}{2} e^{i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix}$$

$$\text{and } |p\rangle_a = (\langle p|_a)^\dagger = \sqrt{2}E \begin{pmatrix} -\sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix}$$

$$P_{ab} = -|p\rangle_a \langle p|_b$$

$$= -2E \begin{pmatrix} -\sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} -\sin \frac{\theta}{2} e^{i\phi} & \cos \frac{\theta}{2} \end{pmatrix} = -2E \begin{pmatrix} \sin^2 \frac{\theta}{2} & -\cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ -\cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & \cos^2 \frac{\theta}{2} \end{pmatrix}$$

$$= 2E \begin{pmatrix} -\sin^2 \frac{\theta}{2} & \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & -\cos^2 \frac{\theta}{2} \end{pmatrix}$$

✓ Yes!

$$-|p\rangle^i \langle p|_b$$

$$= -2E \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix} \begin{pmatrix} \cos \frac{\theta}{2} & \sin \frac{\theta}{2} e^{-i\phi} \end{pmatrix} = -2E \begin{pmatrix} \cos^2 \frac{\theta}{2} & \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \sin \frac{\theta}{2} e^{i\phi} & \sin^2 \frac{\theta}{2} \end{pmatrix}$$

$$= P^{ab} \quad \checkmark \text{ Yes!}$$

2.2 Helicity operator $\Sigma = \frac{\mathbf{S} \cdot \vec{p}}{|\vec{p}|}$

Spin $S_i = \frac{1}{2} \epsilon_{ijk} S^{jk}$, $S^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu]$

If p^μ is along the z-axis, $p^\mu = (E, 0, 0, E)$

$|p \uparrow\rangle = \sqrt{2E} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2E} \\ 0 \end{pmatrix}$

Wb: $\Sigma V_\pm = -h_\pm V_\pm$ for $h_\pm = \pm \frac{1}{2}$

$\Sigma V_+ = -h_+ V_+$
 $= -\frac{1}{2} V_+$

$\Sigma \cdot \begin{pmatrix} |p \uparrow\rangle_a \\ 0 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} |p \uparrow\rangle_a \\ 0 \end{pmatrix}$

$\mathbf{S} \cdot \vec{p} = \mathbf{S} \cdot (0, 0, E) = \frac{1}{2} \epsilon_{ijk} S^{jk} \cdot \vec{E}$
 $= \frac{1}{2} (\epsilon_{312} S^{12} + \epsilon_{321} S^{21}) \cdot \vec{E}$

$= \frac{1}{2} (S^{12} - S^{21}) \cdot \vec{E} = \frac{E}{2} \left(\frac{i}{4} [\gamma^1, \gamma^2] - \frac{i}{4} [\gamma^2, \gamma^1] \right)$

$= \frac{E}{4} i [\gamma^1, \gamma^2]$

$= \cancel{\frac{E}{4} i [\gamma^1, \gamma^2]} \quad \text{p169 Schatz}$
 $= E \sigma_{12}$

$= E \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} \begin{pmatrix} |p \uparrow\rangle_a \\ 0 \end{pmatrix}$

Ex 2.4 $\langle k | \gamma^\mu | k \rangle = 2k^\mu$

$$(0, \langle k | \vec{\alpha} | k \rangle) \begin{pmatrix} 0 & (\bar{\sigma}^\mu)_{ab} \\ (\bar{\sigma}^\mu)_{ab} & 0 \end{pmatrix} \begin{pmatrix} |k\rangle_b \\ 0 \end{pmatrix}$$

$$= (\langle k | \vec{\alpha} (\bar{\sigma}^\mu)_{ab} | k \rangle_b \quad 0) \begin{pmatrix} |k\rangle_b \\ 0 \end{pmatrix}$$

$$= \langle k | \vec{\alpha} (\bar{\sigma}^\mu)_{ab} | k \rangle_b$$

remember:

$$K_{ab} = (K_\mu) (\sigma^\mu)_{ab}$$

$$K^{ab} = K_\mu (\bar{\sigma}^\mu)^{ab}$$

$$= (\langle k | \vec{\alpha} | k \rangle_b, -\langle k | \vec{\alpha} (\sigma^1)^{ab} | k \rangle_b, -\langle k | \vec{\alpha} (\sigma^2)^{ab} | k \rangle_b, -\langle k | \vec{\alpha} (\sigma^3)^{ab} | k \rangle_b)$$

$$\langle K | \cdot K \rangle$$

$$K^{ab} [(\bar{\sigma}^\mu)_{ab}]^{-1} = K_\mu$$

$$K_{ab} [(\sigma^\mu)^{ab}]^{-1} = K_\mu$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$-\langle k | \vec{\alpha} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} | k \rangle_b = -(\langle k |_2 | k \rangle_1 + \langle k |_1 | k \rangle_2)$$

def: $-|k\rangle_a \langle k|_b = K_{ab}$

For real momenta, $[k]^\mu = (|k\rangle^\mu)^\dagger$ so $\langle k | \cdot | k \rangle = \langle k | \cdot (|k\rangle)^\dagger = |k|^2$

$$= \epsilon_{ac} |k\rangle^c (\bar{\sigma}^\mu)^{ab} [k]_b^d \epsilon_{bd}$$

$$= \epsilon_{ac} \epsilon_{bd} (-\delta^{cd}) (\bar{\sigma}^\mu)^{ab}$$

SO: new basis of σ is

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\sigma = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\sigma = \frac{1}{2} \left(\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right)$$

$$\sigma = \frac{1}{2} (1 - \sigma) - \frac{1}{2} \sigma$$

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$$\sigma = 1 - \sigma$$

$$\frac{1}{2} = \sigma$$

$$\sigma = \frac{1}{2} - \frac{1}{2} \sigma$$

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2nd part: $\langle k | P | k \rangle = 2P \cdot K$

" $P_\mu \langle k | \gamma^\mu | k \rangle$

" $P_\mu \cdot 2K^\mu$ by part a)

" $2P \cdot K$

$P_\mu \cdot \langle k | a (\bar{\sigma}^\mu) ab | k \rangle_b$

$= \langle k | a p^{ab} | k \rangle_b$

Goal: project out the $p^0, p^1, \dots$ in Eq 2.7 (should be easy to do if we use explicit formula for $|p\rangle$ and $\langle p|$)

First: $\langle k | \gamma^\mu | k \rangle \langle q | \gamma^\mu | q \rangle = 2 \langle k q \rangle [k q] = 2(p \cdot q) = 4K \cdot q$

Goal: use one of the $\sigma\sigma$ identities

$P_{ab} p_{ab} \cdot \sigma \sim P_{ab} \sigma \cdot \sigma$

$\sigma \cdot \sigma = \epsilon A$

$S \leq 0$ for

$\langle S | \sigma \cdot \sigma | S \rangle = \langle S | \epsilon A | S \rangle = \epsilon \langle S | A | S \rangle$

$\langle S | \sigma \cdot \sigma | S \rangle = \langle S | \epsilon A | S \rangle = \epsilon \langle S | A | S \rangle$

$\langle S | \sigma \cdot \sigma | S \rangle = \langle S | \epsilon A | S \rangle = \epsilon \langle S | A | S \rangle$

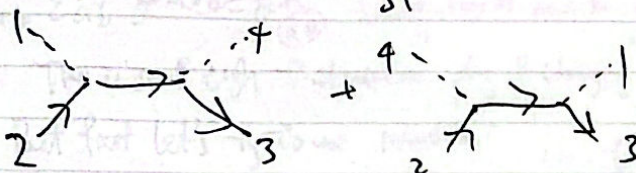
$S = 0, 1, 2, \dots$ but not all $S \leq$ possible values

$n \cdot P$ minimum value for $\epsilon = 0$ or all p values $n \cdot 2$ or S possible values

$\sigma + [2] = 1 \Rightarrow \sigma = 1 - [2] = 1 - 2 = -1$

$\rightarrow [2] \text{ or } 0 - 1 = -1$

$$2.S \langle |A_{\mu}(\phi \bar{\psi} \psi)|^2 \rangle = 2g^4 \frac{(s-t)^2}{st}$$



$$M = (ig)^2 \bar{u}_3 \frac{\not{\epsilon}_1 \not{\epsilon}_2}{(p_1 + p_2)^2} v_2$$

$$\text{so } M = g^2 \bar{u}_3 \frac{\not{p}_1 + \not{p}_2}{(p_1 + p_2)} v_2$$

$$|M|^2 = g^4 \bar{u}_3 \not{p}_1 v_2$$

$$= g^4 \frac{\bar{u}_3 \not{p}_1 v_2 \bar{v}_2 \not{p}_1 u_3}{s}$$

$$\bar{u}_3 \not{p}_2 v_2 \bar{v}_2 \not{p}_2 u_3 + \bar{u}_3 \not{p}_4 v_2 \bar{v}_2 \not{p}_4 u_3 + \bar{u}_3 \not{p}_2 v_2 \bar{v}_2 \not{p}_2 u_3$$

$$+ \text{tr} \left[\not{p}_2 (\not{p}_2 - m) \not{p}_2 (\not{p}_3 + m) \right] + \text{tr} \left[\not{p}_2 (\not{p}_2 - m) \not{p}_2 (\not{p}_3 + m) \right]$$

$$+ \text{tr} \left[\not{p}_4 (\not{p}_2 - m) \not{p}_4 (\not{p}_3 + m) \right] + \text{tr} \left[\not{p}_2 (\not{p}_2 - m) \not{p}_2 (\not{p}_3 + m) \right]$$

$$\text{Actually, I think } |M|^2 \text{ is } \left| g^2 \bar{u}_3 (\not{p}_1 + \not{p}_2) v_2 + g^2 \bar{u}_3 (\not{p}_4 + \not{p}_2) v_2 \right|^2$$

$$= (A+B)(A+B)^*$$

$$= AA^* + BB^* + AB^* + BA^*$$

$$= \frac{g^4}{s^2} \bar{u}_3 (\not{p}_1 + \not{p}_2) v_2 \bar{v}_2 (\not{p}_1 + \not{p}_2) u_3 + \frac{g^4}{u^2} \bar{u}_3 (\not{p}_4 + \not{p}_2) v_2 \bar{v}_2 (\not{p}_4 + \not{p}_2) u_3$$

$$+ \frac{g^4}{su} \bar{u}_3 (\not{p}_1 + \not{p}_2) v_2 \bar{v}_2 (\not{p}_4 + \not{p}_1) u_3 + \frac{g^4}{us} \bar{u}_3 (\not{p}_4 + \not{p}_2) v_2 \bar{v}_2 (\not{p}_1 + \not{p}_2) u_3$$

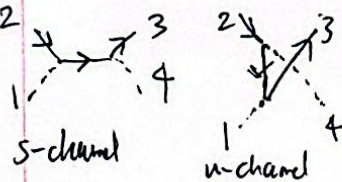
In QFT, V has negative frequency $p^0 < 0$ vs u has positive frequency

$$\text{So } \left(\sum_s u_p^s \bar{u}_p^s = \not{p} - m, \sum_s v_p^s \bar{v}_p^s = \not{p} - m \right)$$

Does this still hold in the notation of B.V.G. - Weyl? (can apply crossing symmetry)

average over many spins/polarizations helicity
sum over outgoing spins/polarizations helicity
Deep ISZ problem - but by crossing symmetry helicity can be switched with outgoing so you have an extra $\frac{1}{4}$ factor?

$A_4(\phi\phi\phi\phi)$
1234



$$|M|^2 = M M^*$$

$$M^* = \bar{M} = \overline{u A v} = \bar{v} A u \text{ where } A = ig^2 \left(\frac{p_1 + p_2}{(p_1 + p_2)^2} + \frac{p_3 + p_4}{(p_3 + p_4)^2} \right)$$

See (2.36) $(ig)^2 \bar{u}_3 - i(p_1 + p_2) \gamma_5 + (1 \leftrightarrow 4)$

so $|M|^2 = M M^* = +g^4 (\bar{u}_3 A v_2) (\bar{v}_2 \bar{A} u_3)$, $\bar{A} = A = \frac{p_1 + p_2}{(p_1 + p_2)^2} + \frac{p_3 + p_4}{(p_3 + p_4)^2}$ since $\gamma^5 = \gamma^5$

Reduction p 291

$$= \sum_{\alpha\beta\gamma\delta} (-g^4) (\bar{u}_3)_\alpha A_{\alpha\beta} (v_2)_\beta (\bar{v}_2)_\gamma A_{\gamma\delta} (u_3)_\delta$$

$$= \sum_{\alpha\beta\gamma\delta} (u_3)_\delta (\bar{u}_3)_\alpha A_{\alpha\beta} (v_2)_\beta (\bar{v}_2)_\gamma A_{\gamma\delta}$$

$$= \text{Tr}[(u_3 \bar{u}_3) A (v_2 \bar{v}_2) A]$$

Phys 110
chap 12

$$\sum_{h=\pm} \sum_{\text{final}} \Rightarrow \langle |M|^2 \rangle = \sum_{S_2 S_3} |M|^2 = \text{Tr}[(\not{p}_3 + m) A (\not{p}_2 + m) A]$$

$$= \text{Tr}[\not{p}_3 A \not{p}_2 A + m A m - \not{p}_3 A m - m A \not{p}_2 A]$$

$$\text{Tr}(m^2 A A) = m^2 \text{Tr} \left[\frac{(p_1 + p_2)^2}{(p_1 + p_2)^4} + 2 \frac{(p_1 + p_3)(p_2 + p_4)}{(p_1 + p_2)^2 (p_3 + p_4)^2} + \frac{(p_3 + p_4)^2}{(p_3 + p_4)^4} \right]$$

$$m^2 \text{Tr} \left[\frac{p_1^2 + 2p_1 p_2 + p_2^2}{(p_1 + p_2)^4} \right] + \frac{2m^2 \text{Tr} [p_1 p_2 + p_2 p_1 + p_1 p_4 + p_4 p_1]}{(p_1 + p_2)^2 (p_3 + p_4)^2} + \frac{m^2 \text{Tr} [p_3^2 + 2p_3 p_4 + p_4^2]}{(p_3 + p_4)^4} = \bar{v}$$

$\text{Tr}(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}$ e.g. $\text{Tr}(p_1 p_2) = \text{Tr}(p_1^\mu \gamma_\mu p_2^\nu \gamma_\nu) = p_1^\mu p_2^\nu \text{Tr}(\gamma_\mu \gamma_\nu) = p_1^\mu p_2^\nu 4g_{\mu\nu} = 4p_1 \cdot p_2$

$$= m^2 \left[\frac{p_1^2 + 2p_1 p_2 + p_2^2}{(p_1 + p_2)^4} + \frac{2(p_1^2 + p_1 p_2 + p_1 p_4 + p_2 p_4)}{(p_1 + p_2)^2 (p_3 + p_4)^2} + \frac{p_3^2 + 2p_3 p_4 + p_4^2}{(p_3 + p_4)^4} \right]$$

$$\text{Tr}[\not{p}_3 A \not{p}_2 A] = \text{Tr} \left[\not{p}_3 \left(\frac{p_1 + p_2}{(p_1 + p_2)^2} + \frac{p_3 + p_4}{(p_3 + p_4)^2} \right) \not{p}_2 \left(\frac{p_1 + p_2}{(p_1 + p_2)^2} + \frac{p_3 + p_4}{(p_3 + p_4)^2} \right) \right]$$

$$= \text{Tr} \left[\not{p}_3 \left(\frac{p_1 + p_2}{(p_1 + p_2)^2} \right) \not{p}_2 \left(\frac{p_1 + p_2}{(p_1 + p_2)^2} \right) + \not{p}_3 \left(\frac{p_1 + p_2}{(p_1 + p_2)^2} \right) \not{p}_2 \left(\frac{p_3 + p_4}{(p_3 + p_4)^2} \right) + \not{p}_3 \left(\frac{p_3 + p_4}{(p_3 + p_4)^2} \right) \not{p}_2 \left(\frac{p_1 + p_2}{(p_1 + p_2)^2} \right) + \not{p}_3 \left(\frac{p_3 + p_4}{(p_3 + p_4)^2} \right) \not{p}_2 \left(\frac{p_3 + p_4}{(p_3 + p_4)^2} \right) \right]$$

$$4(p_3 p_1 p_2 p_2 + p_3 p_2 p_1 p_2 - p_3 p_2 p_1 p_1)$$

continued on p 78

continued from p9

$$\text{Tr}[\not{x}_3 A \not{x}_2 A + m^2 A^2] \quad A = \frac{\not{p}_1 + \not{p}_2}{(p_1 + p_2)^2} + \frac{\not{p}_2 + \not{p}_4}{(p_2 + p_4)^2}$$

$$\text{or Tr}[(\not{p}_3 + m) A (\not{p}_2 + m) A]$$

$$\textcircled{1} \text{Tr}[(\not{p}_3 + m)(\not{p}_1 + \not{p}_2)(\not{p}_2 + m)(\not{p}_1 + \not{p}_2)]/s^2$$

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

$$s = (p_1 + p_2)^2 = 2 p_1 \cdot p_2 = 2 p_3 \cdot p_4$$

$$t = (p_1 + p_3)^2 = 2 p_1 \cdot p_3 = 2 p_2 \cdot p_4$$

$$u = (p_1 + p_4)^2 = 2 p_1 \cdot p_4 = 2 p_2 \cdot p_3$$

swaps $1 \leftrightarrow 4$ changes: $u \leftrightarrow t$
 $s \leftrightarrow t$

$$\textcircled{2} \text{Tr}[(\not{p}_3 + m)(\not{p}_1 + \not{p}_2)(\not{p}_2 + m)(\not{p}_2 + \not{p}_4)]/st$$

$$\textcircled{3} \text{Tr}[(\not{p}_3 + m)(\not{p}_2 + \not{p}_4)(\not{p}_2 + m)(\not{p}_1 + \not{p}_2)]/st$$

$$\textcircled{4} \text{Tr}[(\not{p}_3 + m)(\not{p}_2 + \not{p}_4)(\not{p}_2 + m)(\not{p}_2 + \not{p}_4)]/t^2$$

$$\textcircled{1} \text{Tr}[(\not{p}_3 \not{p}_1 - \not{p}_3 \not{p}_2 + m \not{p}_1 + m \not{p}_2)(\not{p}_2 + m)(\not{p}_1 + \not{p}_2)]$$

8 terms
odd ones die out

$$(\not{p}_2 \not{p}_1 - \not{p}_2^2 + m \not{p}_1 + m \not{p}_2) + \not{p}_3 \not{p}_1 \not{p}_2 \not{p}_1 + \not{p}_3 \not{p}_1 \not{p}_2^2 + \not{p}_3 \not{p}_2 \not{p}_2 \not{p}_1 + \not{p}_3 \not{p}_2 \not{p}_2^2 + m^2 (\not{p}_1^2 + \not{p}_1 \not{p}_2 + \not{p}_2 \not{p}_1 + \not{p}_2^2)$$

$$4(p_3 p_1 p_2 p_1 + p_3 p_1 p_2 p_2 - p_3 p_2 p_2 p_1 + p_3 p_1 p_2 p_2 + p_3 p_2 p_2 p_2 - p_3 p_2 p_2 p_1)$$

$$+ p_3 p_2 p_2 p_1 + p_3 p_1 p_2 p_2 - p_3 p_2 p_2 p_1 + p_3$$

$$+ p_1^2 + p_2^2 + 2 p_1 \cdot p_2 = 8(p_1 p_2 p_1 p_3 + p_1 p_2)$$

$$= 8(2 p_1 \cdot p_2 p_1 p_3) + 4(2 p_1 \cdot p_2)$$

$$= 2st + 4s$$

The total thing is divided by s^2 : $\frac{2st + 4s}{s^2} = 2 \frac{t}{s} + \frac{4}{s}$

$$\textcircled{1} 2 \frac{s}{t} + \frac{4}{t}$$

$$\textcircled{2} (\not{p}_3 \not{p}_1 - \not{p}_3 \not{p}_2 + m \not{p}_1 + m \not{p}_2)(\not{p}_2^2 - \not{p}_2 \not{p}_4 + m \not{p}_2 + m \not{p}_4)$$

$$= p_3 p_1 p_2^2 + p_3 p_1 p_2 p_4 + p_3 p_2 p_2^2 + p_3 p_2 p_2 p_4 + m(p_1 p_2 + p_1 p_4 + p_2^2 + p_2 p_4)$$

$$4(p_3 p_1 p_2 p_2 + p_3 p_2 p_2 p_2 - p_3 p_2 p_2 p_2 + p_3 p_1 p_2 p_4 + p_3 p_4 p_1 p_2 - p_3 p_2 p_1 p_4)$$

$$+ p_3 p_2 p_2 p_4 + p_3 p_1 p_2 p_2 - p_3 p_2 p_2 p_4 + m^2(p_1 p_2 + p_1 p_4 + p_2 p_4)$$

$$= 4(p_1 p_2 p_2 p_4 + p_1 p_2 p_3 p_4 - p_2 p_3 p_1 p_4) + m^2(p_1 p_2 + p_1 p_4 + p_2 p_4)$$

$$= t^2 + s^2 - u^2 + 2m(s + u + t)$$

$\oint \gamma = \sum m_j^2$ but assume all particles massless

divide by st : $\frac{t^2 + s^2 - u^2}{st}$

$$(3) \frac{t^2 + s^2 - u^2}{st} \text{ as well}$$

so all together:

$$2 \frac{s}{t} + 2 \frac{t}{s} + \frac{2}{st} (t^2 + s^2 - u^2)$$

$$= \frac{2s^2 + 2t^2 + 2t^2 + 2s^2 - 2u^2}{st}$$

$$= \frac{4s^2 + 4t^2 - 2u^2}{st}$$

$$= 2 \cdot \frac{2s^2 + 2t^2 - u^2}{st} \text{ but } s+t+u=0$$

$$u = -s-t$$

$$u^2 = s^2 + 2st + t^2$$

$$= 2 \cdot \frac{2s^2 + 2t^2 - (s^2 + 2st + t^2)}{st}$$

$$= 2 \cdot \frac{s^2 + t^2 - 2st}{st} = \frac{2(s-t)^2}{st}$$

Remainder factor of q from p :

$$E_{2q} \left(\frac{s^2 + t^2 - 2st}{st} \right) + 2q \frac{(s-t)^2}{st} =$$

$$= \frac{E_{2q}(s^2 + t^2 - 2st) + 2q(s-t)^2}{st}$$

$$= \frac{E_{2q}(s^2 + t^2 - 2st) + 2q(s-t)^2}{st}$$

$$= \frac{E_{2q}(s^2 + t^2 - 2st) + 2q(s-t)^2}{st}$$

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$$= \frac{E_{2q}(s^2 + t^2 - 2st) + 2q(s-t)^2}{st}$$

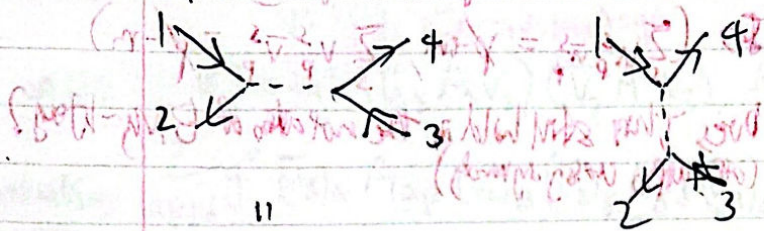
$$= \frac{E_{2q}(s^2 + t^2 - 2st) + 2q(s-t)^2}{st}$$

$$= \frac{E_{2q}(s^2 + t^2 - 2st) + 2q(s-t)^2}{st}$$

$$= \frac{E_{2q}(s^2 + t^2 - 2st) + 2q(s-t)^2}{st}$$

$$= \frac{E_{2q}(s^2 + t^2 - 2st) + 2q(s-t)^2}{st}$$

2.6 $A_q(\bar{p}^+ p^+ \bar{p}^- p^-)$ where we consider exactly outgoing $1, 2, 3, 4$



$$ig \bar{u}_4 v_3 \frac{-i}{(p_1 + p_2)^2} ig \bar{u}_2 v_1$$

$$ig \bar{u}_4 v_1 \frac{-i}{(p_1 + p_4)^2} ig \bar{u}_2 v_3$$

$$(0, \langle 4 |) \begin{pmatrix} 0 \\ 3 \rangle \end{pmatrix}$$

$$ig^2 \frac{\langle 43 \rangle \langle 21 \rangle}{[12]}$$

$$ig^2 \frac{\langle 41 \rangle \langle 23 \rangle}{[14]}$$

$$-ig^2 \frac{\langle 43 \rangle}{[12]} + -ig^2 \frac{\langle 23 \rangle}{[14]}$$

$$\langle 43 \rangle [32] + \langle 43 \rangle [31] = 0 \rightarrow \langle 43 \rangle = -\frac{\langle 41 \rangle [12]}{[32]}$$

$$\text{or } \langle 43 \rangle [31] + \langle 42 \rangle [21] = 0$$

$$-ig^2 \frac{1}{[12]} \left(-\frac{\langle 41 \rangle [12]}{[32]} \right) = ig^2 \frac{\langle 41 \rangle}{[32]}$$

$$ig^2 \left(\frac{\langle 41 \rangle}{[32]} - \frac{\langle 23 \rangle}{[14]} \right)$$

$$= ig^2 \left(\frac{\langle 41 \rangle [14] - \langle 23 \rangle [32]}{[32][14]} \right)$$

$$= ig^2 \left(\frac{\langle 23 \rangle [23] - \langle 14 \rangle [14]}{[14][32]} \right)$$

$$= 2ig^2 \left(\frac{p_2 \cdot p_3 - p_1 \cdot p_4}{[14][32]} \right)$$

$$= 2ig^2 \left(\frac{p_1 \cdot p_4 - p_2 \cdot p_3}{[14][23]} \right)$$

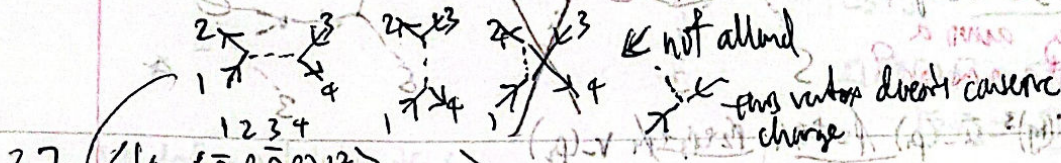
$p_1 + p_2 + p_3 + p_4 = 0$ by momentum conservation

$$p_1 + p_4 = -p_2 - p_3$$

$$(p_1 + p_4)^2 = (p_2 + p_3)^2$$

$$2p_1 \cdot p_4 = 2p_2 \cdot p_3$$

$$= 0$$



2.7 $\langle |A_q(\bar{p} + \bar{p} f)|^2 \rangle$

$$(ig)^2 \bar{u}_2 V_1 \frac{-i}{(p_1 + p_2)^2} \bar{u}_4 V_3$$

$$(ig)^2 \bar{u}_2 V_3 \frac{-i}{(p_1 + p_4)^2} \bar{u}_4 V_1$$

This diagram not have 1,2 same helicity, 3,4 same helicity

This diagram not have 1,4 same helicity, 2,3 same helicity

$$A_q(++++) + A_q(+-+-) + A_q(----) + A_q(--++)$$

$$A_q(+-+-) + A_q(----) + A_q(-++-) + A_q(-+-+)$$

so we have $\langle A_q^2 \rangle = |A_q(++++)|^2 + |A_q(----)|^2 + |A_q(+-+-)|^2 + |A_q(-++-)|^2 + |A_q(-+-+)|^2 + |A_q(--++)|^2$

there are 2 terms:

$$\left| \underbrace{A_q(++++)}_{\text{diagram 1 s-channel}} + \underbrace{A_q(+-+-)}_{\text{diagram 2 u-channel}} \right|^2$$

from p7: $A_q(----) = 0$

$$A_q(++++) = (ig)^2 [21] \frac{-i}{(p_1 + p_2)^2} [43] + (ig)^2 [23] \frac{-i}{(p_1 + p_4)^2} [41]$$

$$= ig^2 \frac{[21][43]}{[21][21]} + ig^2 \frac{[23][41]}{[41][41]}$$

$$= ig^2 \left(\frac{[43]}{[21]} + \frac{[23]}{[41]} \right) \quad [23]\langle 31 \rangle + [41]\langle 41 \rangle = 0$$

$$= ig^2 \left(\frac{[43]}{[21]} + \frac{\langle 14 \rangle [43]}{\langle 12 \rangle \langle 41 \rangle} \right) \quad [23] = -\frac{[21]\langle 41 \rangle}{\langle 31 \rangle}$$

$$= ig^2 \left(\frac{[43]}{[21]} + \frac{[43]}{[12]} \right) \quad \langle 12 \rangle [23] + \langle 14 \rangle [43] = 0$$

$$[23] = -\frac{\langle 14 \rangle [43]}{\langle 12 \rangle}$$

Diagram 1: $A_q(+-+-) = ig^2 [21] \frac{-i}{(p_1 + p_2)^2} \langle 43 \rangle = ig^2 \frac{[21]\langle 43 \rangle}{[21][21]} = ig^2 \frac{\langle 43 \rangle}{[21]}$

$$A_q(--++) = ig^2 [21] \frac{-i}{(p_1 + p_4)^2} [43] = ig^2 \frac{[43]}{[21]}$$

Diagram 2: $A_q(+-+-) = ig^2 [23] \frac{-i}{(p_1 + p_4)^2} [41] = ig^2 \frac{\langle 23 \rangle}{[41]}$

$$A_q(-++-) = ig^2 [23] \frac{-i}{(p_1 + p_2)^2} [41] = ig^2 \frac{[23]}{[41]}$$



$$\text{Then, } |A_4(+ + -)|^2 + |A_4(- + +)|^2 + |A_4(+ - +)|^2 + |A_4(- + -)|^2$$

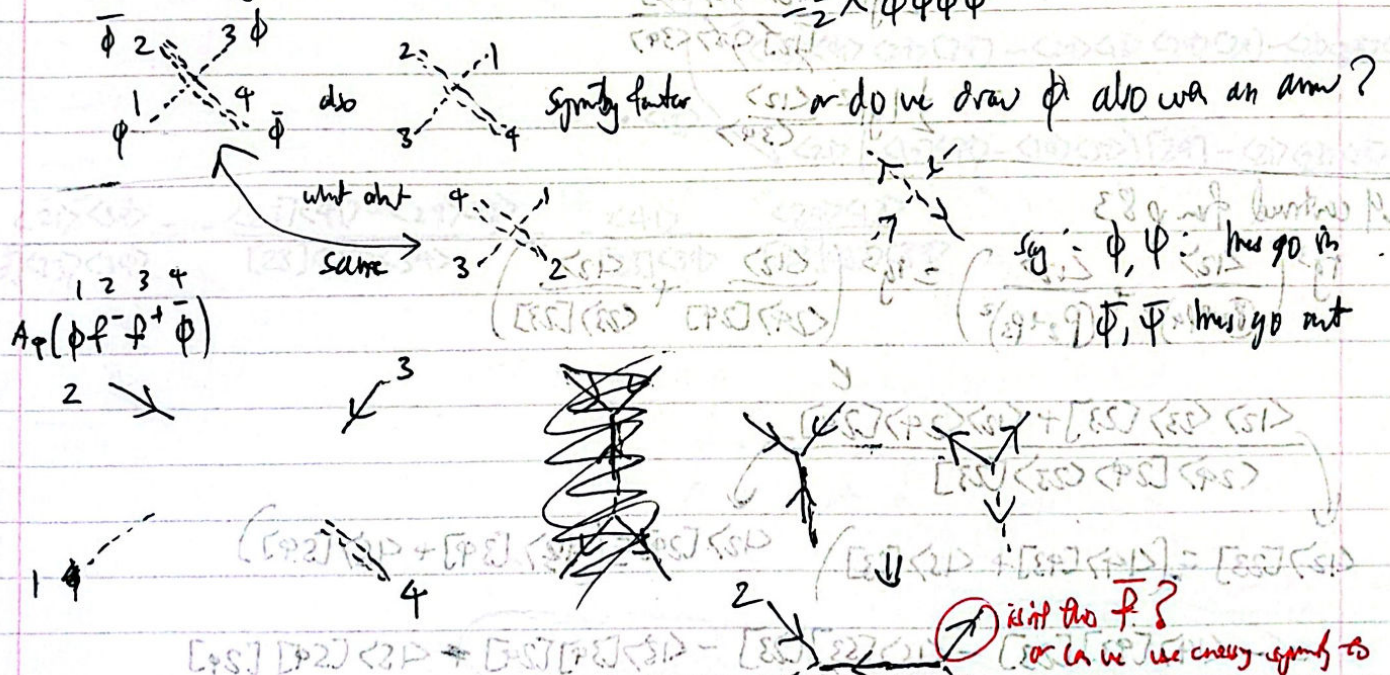
$$= g^4 \left(\frac{\langle 43 \rangle \langle 43 \rangle}{\langle 21 \rangle \langle 21 \rangle} + \frac{\langle 43 \rangle \langle 43 \rangle}{\langle 21 \rangle \langle 21 \rangle} + \frac{\langle 23 \rangle \langle 23 \rangle}{\langle 41 \rangle \langle 41 \rangle} + \frac{\langle 23 \rangle \langle 23 \rangle}{\langle 41 \rangle \langle 41 \rangle} \right)$$

$$= 2g^4 \left(\frac{\langle 43 \rangle \langle 43 \rangle}{\langle 21 \rangle \langle 21 \rangle} + \frac{\langle 23 \rangle \langle 23 \rangle}{\langle 41 \rangle \langle 41 \rangle} \right)$$
 and $\langle 43 \rangle \langle 43 \rangle = (p_3 + p_4)^2 = (p_1 + p_2)^2 = \langle 21 \rangle \langle 21 \rangle$ by momentum conservation

$$= 2g^4(1+1)$$

$$= 4g^4$$

2.8 $\mathcal{L} = \psi^\dagger \sigma^\mu \partial_\mu \psi - \partial_\mu \bar{\psi} \partial^\mu \psi + \frac{1}{2} g \phi \psi \psi + \frac{1}{2} g \bar{\psi} \bar{\psi} \phi - \frac{1}{2} \lambda |\phi|^4$
 $A_4(\phi \phi \bar{\psi} \bar{\psi})$, if ϕ : $\phi \cdots \begin{matrix} \psi \\ \psi \end{matrix}$ but no loop so each close up, and no $\bar{\psi}$ vertices exist, so $\bar{\psi}$ in any is wrong
 so the only ~~diagram~~ with 4 ϕ 's is the 4-vertex: $-\frac{1}{2} \lambda \phi \bar{\phi} \phi \bar{\phi}$



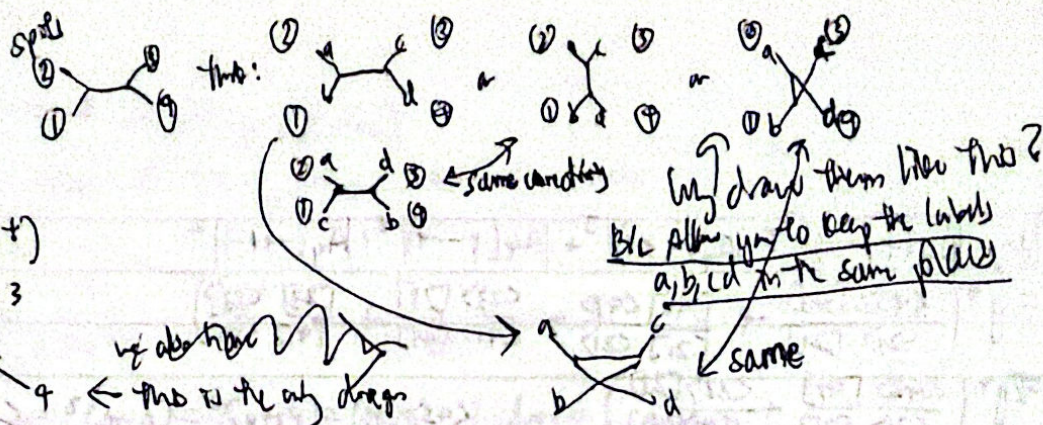
$$A_4(\phi \phi \bar{\psi} \bar{\psi})$$

$$= \frac{1}{4} g^2 \frac{\bar{u}_2 \gamma^\mu \sigma^\mu (p_1 + p_2)_\mu V_3}{(p_1 + p_2)^2} = \frac{1}{4} g^2 \frac{\bar{u}_2 \gamma^\mu \sigma^\mu (p_1 + p_2)_\mu V_3}{(p_1 + p_2)^2}$$

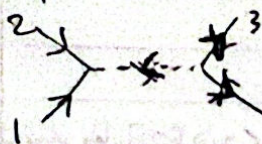
$$= \frac{1}{4} g^2 \left(\frac{\bar{u}_2 \gamma^\mu \sigma^\mu (p_1)_\mu V_3}{(p_1 + p_2)^2} + \frac{\bar{u}_2 \gamma^\mu \sigma^\mu (p_2)_\mu V_3}{(p_1 + p_2)^2} \right) = \frac{1}{4} g^2 \left(\frac{\langle 21 \rangle \langle 13 \rangle}{(p_1 + p_2)^2} + \frac{\langle 22 \rangle \langle 23 \rangle}{(p_1 + p_2)^2} \right)$$

$$= \frac{1}{4} g^2 \frac{\langle 21 \rangle \langle 13 \rangle}{\langle 21 \rangle \langle 21 \rangle} = \frac{1}{4} g^2 \frac{\langle 13 \rangle}{\langle 21 \rangle}$$

$$= \frac{1}{4} g^2 \frac{\langle 24 \rangle \langle 24 \rangle}{\langle 13 \rangle \langle 34 \rangle} \cdot \frac{\langle 21 \rangle}{\langle 34 \rangle \langle 34 \rangle} = \frac{1}{4} g^2 \frac{\langle 24 \rangle \langle 21 \rangle}{\langle 34 \rangle \langle 13 \rangle} = 1$$



$$A_4(p^+ p^+ p^+ p^+)$$



$$\left(\frac{1}{2}g\right)\left(\frac{1}{2}g\right)\left(\bar{u}_1 \bar{u}_2 \frac{1}{(p_1+p_2)^2} v_3 v_4\right) ?$$

$$\bar{u}_2 v_3 \bar{u}_1 v_4 ? \text{ the other possible order would be } \bar{u}_2 v_4 \bar{u}_1 v_3$$

$$\begin{aligned}
 -\frac{1}{2}g^2 \frac{\langle 12 \rangle [34]}{\langle 12 \rangle [12]} &= -\frac{1}{2}g^2 \frac{[34] \langle 12 \rangle [34]}{[12] \langle 12 \rangle [34]} \\
 &= -\frac{1}{2}g^2 \frac{\langle 12 \rangle [34]}{[12] \langle 12 \rangle [34]} \\
 &= -\frac{1}{2}g^2 \frac{\langle 12 \rangle}{[34]}
 \end{aligned}$$

2.9 continued from p 83

$$ig^3 \left(\frac{\langle 12 \rangle}{(p_2+p_4)^2} + \frac{\langle 12 \rangle}{(p_2+p_3)^2} \right) = ig^3 \left(\frac{\langle 12 \rangle}{\langle 24 \rangle [24]} + \frac{\langle 12 \rangle}{\langle 23 \rangle [23]} \right)$$

$$\frac{\langle 12 \rangle \langle 23 \rangle [23] + \langle 12 \rangle \langle 24 \rangle [24]}{\langle 24 \rangle [24] \langle 23 \rangle [23]}$$

$$\langle 12 \rangle [23] = \langle 14 \rangle [93] + \langle 15 \rangle [53] \quad \langle 12 \rangle [24] = \langle 13 \rangle [34] + \langle 15 \rangle [54]$$

$$= -\frac{\langle 14 \rangle [93] [23] - \langle 15 \rangle [53] [23] - \langle 13 \rangle [34] [24] + \langle 15 \rangle [54] [24]}{\langle 24 \rangle [24] \langle 23 \rangle [23]}$$

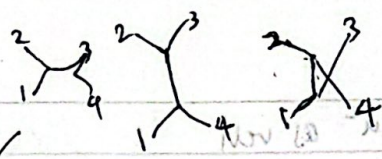
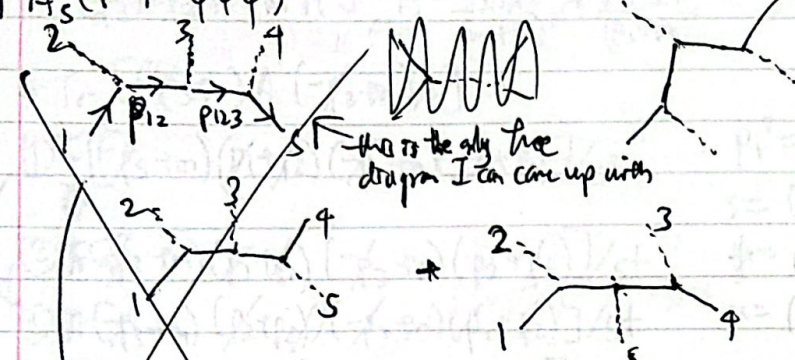
From p 83: other terms:

$$\frac{-\langle 21 \rangle}{(p_2+p_3)^2} - \frac{\langle 21 \rangle}{(p_2+p_4)^2} + \frac{[23] [43] \langle 13 \rangle + [24] [34] \langle 14 \rangle}{[23] [24] A}$$

$$\frac{\langle 21 \rangle}{\langle 23 \rangle [23]} = \frac{\langle 21 \rangle}{\langle 24 \rangle [24]} + \frac{[25] [34]}{[23] [24] [51]}$$

$$= \frac{\langle 21 \rangle \langle 24 \rangle [24] [51] - \langle 21 \rangle \langle 23 \rangle [23] [51] + [25] [34] \langle 23 \rangle \langle 24 \rangle}{\langle 23 \rangle [23] \langle 24 \rangle [24] [51]}$$

2.9 $A_5(p, \bar{p}, \phi, \phi, \phi)$

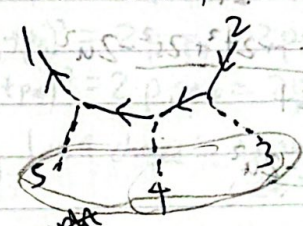


~~$$(ig)^3 \bar{u}_5 \frac{i(p_1 + p_2)}{(p_1 + p_2)^2} - i \frac{(p_1 + p_2 + p_3)}{(p_1 + p_2 + p_3)^2} V_1$$

$$(-i)^2 (ig)^3 \langle 5 | (1+2)(1+2+3) | 1 \rangle$$

$$p_1 p_1 + 2 p_1 p_2 + p_1 p_3 + p_2 p_3$$~~

1 2 3 4 5
 $\bar{p} \phi \phi \phi$



Seems like there should be 3! = 6 different permutations/diagrams why are there only 3?

435
453
543
534
345
354

$$(-i)^2 (ig)^3 \bar{u}_5 - (p) \frac{p_2 + p_3}{(p_2 + p_3)^2} \frac{(p_2 + p_3 + p_4)}{(p_2 + p_3 + p_4)^2} V_2(p_2)$$

$$(0, \langle p_1 a |) \begin{pmatrix} 0 & (\sigma^{\mu})_{ab} \\ (\bar{\sigma}^{\mu})_{ab} & 0 \end{pmatrix} \begin{pmatrix} 0 & (\sigma^{\nu})_{ab} \\ (\bar{\sigma}^{\nu})_{ab} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 p_2^a \end{pmatrix}$$

$$(\langle p_1 a | (\bar{\sigma}^{\mu})_{ab} 0) ((\sigma^{\nu})_{ca} | p_2^d)$$

$$= \langle p_1 a | (\bar{\sigma}^{\mu})_{ab} (\sigma^{\nu})_{bc} | p_2^c \rangle$$

anyway p_2 to the right dies, so we have

~~$$p_2 p_3 + p_2 p_4 + p_3 p_4$$~~

eg. $\langle p_1 p_2 p_3 | p_2 \rangle$
 $= \langle p_1 a | p_2^a (\bar{\sigma}^{\mu})_{ab} p_3^b (\sigma^{\nu})_{bc} | p_2^c \rangle$
 $= \langle p_1 a | (p_2)^{ab} (p_3)_{bc} | p_2^c \rangle$

$$\langle 12 \rangle (\langle 23 \rangle \langle 32 \rangle + \langle 24 \rangle \langle 42 \rangle + \langle 34 \rangle \langle 43 \rangle) - \langle 14 \rangle \langle 23 \rangle \langle 34 \rangle$$

$\langle 14 \rangle \langle 23 \rangle = \langle 12 \rangle \langle 34 \rangle$ by Schouten

~~$$= \langle p_1 | p_2 \rangle \langle p_2 p_3 \rangle \langle p_3 p_2 \rangle$$

$$= \langle 12 \rangle \langle 23 \rangle \langle 32 \rangle + \langle 12 \rangle \langle 24 \rangle \langle 42 \rangle + \langle 13 \rangle \langle 34 \rangle \langle 42 \rangle$$

$$\frac{(p_2 + p_3)^2 (p_2 + p_3 + p_4)^2}{\langle 23 \rangle \langle 23 \rangle}$$

$$2 p_1 p_2 + 2 p_1 p_4 + 2 p_3 p_4$$

$$\langle 23 \rangle \langle 23 \rangle + \langle 24 \rangle \langle 24 \rangle + \langle 34 \rangle \langle 34 \rangle$$~~

$$= \langle 42 \rangle (\langle 12 \rangle \langle 24 \rangle + \langle 13 \rangle \langle 34 \rangle)$$

(= $\langle 15 \rangle \langle 54 \rangle$) by momentum conservation: see next page

page 82

$$0 = \langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 24 \rangle + \langle 14 \rangle \langle 23 \rangle$$

$$\langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 24 \rangle + \langle 14 \rangle \langle 23 \rangle = \langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 24 \rangle + \langle 14 \rangle \langle 23 \rangle$$

$$0 = \langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 24 \rangle + \langle 14 \rangle \langle 23 \rangle$$

$$p_1 + p_2 + p_3 + p_4 + p_5 = 0$$

$$\sum_i |i\rangle \langle i| = 0$$

$$\langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 24 \rangle + \langle 14 \rangle \langle 23 \rangle = 0$$

$$\langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 24 \rangle + \langle 14 \rangle \langle 23 \rangle = 0$$

$$\langle 12 \rangle \langle 34 \rangle - \langle 13 \rangle \langle 24 \rangle - \langle 14 \rangle \langle 23 \rangle$$

$$\text{determinant can also be written: } \langle 23 \rangle \langle 23 \rangle \langle 15 \rangle \langle 15 \rangle$$

$$\rightarrow \frac{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle}{\langle 23 \rangle \langle 23 \rangle \langle 15 \rangle \langle 15 \rangle} - \frac{\langle 42 \rangle \langle 45 \rangle \langle 54 \rangle}{\langle 23 \rangle \langle 23 \rangle \langle 15 \rangle \langle 15 \rangle}$$

$$= - \frac{\langle 12 \rangle}{\langle 15 \rangle \langle 15 \rangle} - \frac{\langle 42 \rangle \langle 54 \rangle}{\langle 15 \rangle \langle 23 \rangle \langle 23 \rangle}$$

$$= - \frac{\langle 12 \rangle}{\langle 15 \rangle \langle 15 \rangle} + \frac{\langle 53 \rangle \langle 34 \rangle}{\langle 15 \rangle \langle 23 \rangle \langle 23 \rangle} + \frac{\langle 51 \rangle \langle 42 \rangle}{\langle 15 \rangle \langle 23 \rangle \langle 23 \rangle}$$

$$= - \left(\frac{\langle 12 \rangle}{\langle 15 \rangle \langle 15 \rangle} + \frac{\langle 12 \rangle}{\langle 23 \rangle \langle 23 \rangle} + \frac{\langle 53 \rangle}{\langle 15 \rangle \langle 23 \rangle} \right)$$

$$= - \left(\frac{\langle 12 \rangle \langle 23 \rangle \langle 23 \rangle + \langle 12 \rangle \langle 45 \rangle \langle 15 \rangle + \langle 53 \rangle \langle 15 \rangle \langle 23 \rangle}{\langle 15 \rangle \langle 15 \rangle \langle 23 \rangle \langle 23 \rangle} \right)$$

ply and with Schur:

$$\langle 15 \rangle \langle 23 \rangle + \langle 13 \rangle \langle 52 \rangle + \langle 12 \rangle \langle 35 \rangle = 0$$

$$\langle 12 \rangle \langle 23 \rangle + \langle 13 \rangle \langle 22 \rangle + \langle 12 \rangle \langle 32 \rangle = 0 \text{ clearly since } \langle 12 \rangle \langle 23 \rangle = - \langle 12 \rangle \langle 32 \rangle$$

$$= - \langle 12 \rangle \langle 35 \rangle - \langle 13 \rangle \langle 52 \rangle$$

$$\text{Numerator: } \langle 12 \rangle (\langle 23 \rangle \langle 23 \rangle + \langle 45 \rangle \langle 15 \rangle - \langle 53 \rangle \langle 35 \rangle + \langle 25 \rangle \langle 52 \rangle) + \langle 14 \rangle \langle 45 \rangle \langle 52 \rangle$$

$$- (\langle 12 \rangle \langle 23 \rangle \langle 23 \rangle + \langle 45 \rangle \langle 15 \rangle + \langle 35 \rangle \langle 35 \rangle) - \langle 53 \rangle \langle 43 \rangle \langle 52 \rangle$$

$$[34]\langle 41 \rangle + [35]\langle 51 \rangle + [32]\langle 21 \rangle = 0$$

$$[34] = -\frac{[35]\langle 51 \rangle - [32]\langle 21 \rangle}{\langle 41 \rangle}$$

$$[42][34] + [13][42] + [14][23] = 0$$

$$[34] = -\frac{[13][42] - [14][23]}{[12]}$$

$$0 = [12]\langle 41 \rangle + [13]\langle 42 \rangle + [14]\langle 43 \rangle$$

$$0 = [12]\langle 21 \rangle + [13]\langle 22 \rangle + [14]\langle 23 \rangle$$

$$[12]\langle 21 \rangle + [13]\langle 22 \rangle + [14]\langle 23 \rangle = 0$$

What we just did we did:

$$\frac{\langle 12 \rangle [23] \langle 32 \rangle}{\langle 23 \rangle [23] \langle 15 \rangle [15]} = -\frac{\langle 12 \rangle}{\langle 15 \rangle [15]}$$

$$\frac{\langle 12 \rangle [23] \langle 32 \rangle - \langle 12 \rangle \langle 15 \rangle [54]}{\langle 23 \rangle [23] \langle 15 \rangle [15]}$$

$$\langle 41 \rangle \langle 52 \rangle + \langle 43 \rangle \langle 21 \rangle$$

$$\langle 12 \rangle \langle 23 \rangle \langle 32 \rangle + \langle 41 \rangle \langle 52 \rangle [54] + \langle 45 \rangle \langle 53 \rangle \langle 21 \rangle$$

Cont fm p 80

$$\frac{\langle 12 \rangle (\langle 23 \rangle \langle 32 \rangle + \langle 24 \rangle \langle 42 \rangle + \langle 34 \rangle \langle 43 \rangle) - \langle 14 \rangle \langle 23 \rangle [34]}{(\langle 23 \rangle [23]) (\langle 23 \rangle [23] + \langle 24 \rangle [24] + \langle 34 \rangle [34])}$$

$$-\frac{\langle 12 \rangle}{\langle 23 \rangle [23]} - \frac{\langle 14 \rangle \langle 23 \rangle [34]}{\langle 23 \rangle [23] (\langle 23 \rangle [23] + \langle 24 \rangle [24] + \langle 34 \rangle [34])}$$

$$\langle 14 \rangle [34] = -\langle 14 \rangle [43]$$

$$= \langle 12 \rangle [23] + \langle 15 \rangle [53]$$

$$+\frac{\langle 12 \rangle}{\langle 23 \rangle [23] + \langle 24 \rangle [24] + \langle 34 \rangle [34]} + \frac{\langle 15 \rangle [53]}{\langle 23 \rangle [23] + \langle 24 \rangle [24] + \langle 34 \rangle [34]}$$

$$-\frac{\langle 12 \rangle}{\langle 23 \rangle [23]} + \frac{\langle 12 \rangle}{\langle 15 \rangle [15]} + \frac{[53]}{\langle 23 \rangle [15]}$$

$$-\frac{\langle 12 \rangle [15] + [53] \langle 23 \rangle}{\langle 23 \rangle [23] [15]}$$

$$[34] [12] = \langle 12 \rangle [34]$$

$$([13][42] + [14][23]) \langle 14 \rangle \langle 23 \rangle$$

$$\langle 23 \rangle (\langle 12 \rangle [23] + \langle 15 \rangle [53])$$

$$+\frac{\langle 14 \rangle \langle 23 \rangle ([13][42] + [14][23])}{\langle 23 \rangle [23] [12]}$$

See top of p80: add the 3rd dgm:

Since the flip is happy across a fermion line, does it add a minus sign?



$$\begin{aligned}
 & \frac{\langle 12 \rangle}{\langle 23 \rangle [23]} = (-1)^2 (i g^3) \bar{u} = (p_1) \frac{(p_2 + p_4) (p_2 + p_3 + p_4) v = (p_2)}{(p_2 + p_4)^2 (p_2 + p_3 + p_4)^2} \\
 & = (i g^3) \frac{\langle 11 \rangle (p_2 p_3 + p_2 p_4 + p_4 p_3) \langle 2 \rangle}{(p_2 + p_4)^2 (p_2 + p_3 + p_4)^2} \\
 & = i g^3 \frac{(\langle 12 \rangle [23] \langle 32 \rangle + \langle 12 \rangle [24] \langle 42 \rangle + \langle 14 \rangle [43] \langle 32 \rangle)}{(p_2 + p_4)^2 (p_2 + p_3 + p_4)^2} \\
 & = i g^3 \frac{(\langle 12 \rangle ([23] \langle 32 \rangle + [24] \langle 42 \rangle - [43] \langle 43 \rangle) - [43] \langle 13 \rangle \langle 24 \rangle)}{(p_2 + p_4)^2 (p_2 + p_3 + p_4)^2} \\
 & = i g^3 \left(-\frac{\langle 12 \rangle}{(p_2 + p_4)^2} - \frac{[43] \langle 43 \rangle \langle 24 \rangle}{(p_2 + p_4)^2 (p_2 + p_3 + p_4)^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{[43] \langle 43 \rangle \langle 24 \rangle}{\langle 24 \rangle [24] A} + \frac{[34] \langle 44 \rangle \langle 23 \rangle}{\langle 23 \rangle [23] A} \text{ where } A = (p_2 + p_3 + p_4)^2 \\
 & \frac{[23] [43] \langle 43 \rangle + [24] [34] \langle 14 \rangle}{[23] [24] A}
 \end{aligned}$$

$$\begin{aligned}
 [23] \langle 31 \rangle &= -([24] \langle 41 \rangle + [25] \langle 51 \rangle) \\
 [24] \langle 41 \rangle &= +([23] \langle 31 \rangle + [25] \langle 51 \rangle)
 \end{aligned}$$

$$\frac{-[43] \langle 31 \rangle}{[24] A} = \frac{[42] \langle 21 \rangle + [45] \langle 51 \rangle}{[24] A}$$

$$\begin{aligned}
 & [23] \langle 43 \rangle [43] + [24] \langle 14 \rangle [34] \\
 & = -[23] \langle 31 \rangle [43] + [24] \langle 41 \rangle [34] \\
 & = -([23] \langle 31 \rangle - [24] \langle 41 \rangle) [34]
 \end{aligned}$$

could be zero if only 4 particles

$$\frac{-[34] \langle 41 \rangle}{[23] A} = \frac{[32] \langle 21 \rangle + [35] \langle 51 \rangle}{[23] A}$$

$$= -\frac{\langle 21 \rangle}{(p_2 + p_4)^2} + \frac{[45] \langle 51 \rangle}{[24] A} - \frac{\langle 21 \rangle}{(p_2 + p_4)^2} + \frac{[35] \langle 51 \rangle}{[23] A}$$

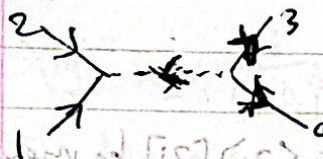
$$\frac{[45] \langle 51 \rangle}{[24] [51]} + \frac{[35] \langle 51 \rangle}{[23] [51]} = \frac{[23] [45] + [24] [35]}{[24] [23] [51]}$$

$$= \frac{([23] [45] + [24] [35])}{[23] [24] [51]}$$

$$= \frac{[45]}{[24] [51]} + \frac{[35]}{[23] [51]}$$

cut on p86

$$A_q(p^- p^- p^+ p^+)$$



$$\left(\frac{1}{2}g\right)\left(\frac{1}{2}g\right)\frac{\bar{u}_1 u_2}{(p_1 + p_2)^2} V_3 V_4$$

$$\bar{u}_2 v_3 \bar{u}_1 v_4 \text{ ? the other possible combo would be } \bar{u}_2 v_4 \bar{u}_1 v_3$$

$$-\frac{1}{2}g^2 \frac{\langle 12 \rangle [34]}{\langle 12 \rangle [12]} = -\frac{1}{2}g^2 \frac{[34] \langle 12 \rangle [34]}{[12] \langle 12 \rangle [34]}$$

$$= -\frac{1}{2}g^2 \frac{\langle 12 \rangle [34] \langle 12 \rangle [34]}{[12] [34] \langle 12 \rangle [34]}$$

$$= -\frac{1}{2}g^2 \frac{\langle 12 \rangle}{[34]}$$

2.9 continued from p 83

$$ig^3 \left(\frac{\langle 12 \rangle}{(p_2 + p_4)^2} + \frac{\langle 12 \rangle}{(p_2 + p_3)^2} \right) = ig^3 \left(\frac{\langle 12 \rangle}{\langle 24 \rangle [24]} + \frac{\langle 12 \rangle}{\langle 23 \rangle [23]} \right)$$

$$\frac{\langle 12 \rangle \langle 23 \rangle [23] + \langle 12 \rangle \langle 24 \rangle [24]}{\langle 24 \rangle [24] \langle 23 \rangle [23]}$$

$$\langle 12 \rangle [23] = \langle 14 \rangle [43] + \langle 15 \rangle [53] \quad \langle 12 \rangle [24] = \langle 13 \rangle [34] + \langle 15 \rangle [54]$$

$$= -\langle 14 \rangle [43] [23] - \langle 15 \rangle [53] [23] - \langle 13 \rangle [34] [24] - \langle 15 \rangle [54] [24]$$

From p 83: other terms:

$$\frac{-\langle 21 \rangle}{(p_2 + p_3)^2} - \frac{\langle 21 \rangle}{(p_2 + p_4)^2} + \frac{[23] [43] \langle 13 \rangle + [24] [34] \langle 14 \rangle}{[23] [24] A}$$

$$\frac{\langle 21 \rangle}{\langle 23 \rangle [23]} = \frac{\langle 21 \rangle}{\langle 24 \rangle [24]} + \frac{[25] [34]}{[23] [24] [51]}$$

$$= \frac{\langle 21 \rangle \langle 24 \rangle [24] [51]}{\langle 23 \rangle [23] \langle 24 \rangle [24] [51]} - \frac{\langle 21 \rangle \langle 23 \rangle [23] [51] + [25] [34] \langle 23 \rangle \langle 24 \rangle}{\langle 23 \rangle [23] \langle 24 \rangle [24] [51]}$$

$$\frac{[12] \langle 21 \rangle}{[12] \langle 23 \rangle [23]}$$

$$\frac{[43] \langle 43 \rangle \langle 29 \rangle}{(\beta_2 + \beta_4)^2 A} - \frac{[34] \langle 49 \rangle \langle 23 \rangle}{(\beta_2 + \beta_3)^2 A}$$

$$- [14] \langle 43 \rangle + [15] \langle 53 \rangle$$

$$\langle 23 \rangle = - \frac{[14] \langle 43 \rangle - [15] \langle 53 \rangle}{[12]}$$

$$[43] (-\langle 12 \rangle \langle 43 \rangle - \langle 14 \rangle \langle 32 \rangle)$$

$$\langle 29 \rangle [29] (\langle 23 \rangle [23] + \langle 29 \rangle [14] + \langle 34 \rangle [34])$$

$$+ \frac{[34] \langle 42 \rangle \langle 43 \rangle + [34] \langle 44 \rangle \langle 32 \rangle + [34] \langle 49 \rangle \langle 23 \rangle}{(\beta_2 + \beta_4)^2 A} - \frac{[34] \langle 49 \rangle \langle 23 \rangle}{(\beta_2 + \beta_3)^2 A}$$

$$\frac{[34] \langle 42 \rangle \langle 43 \rangle \langle 29 \rangle [29] + [34] \langle 44 \rangle \langle 32 \rangle \langle 23 \rangle [23] \pm [34] \langle 49 \rangle \langle 23 \rangle \langle 29 \rangle [24]}{\langle 23 \rangle [23] \langle 29 \rangle [29] A}$$

$$\frac{\langle 21 \rangle \langle 34 \rangle [24] - \langle 21 \rangle \langle 23 \rangle [23]}{\langle 23 \rangle [23] \langle 29 \rangle [24]} \cdot \frac{[34]}{\langle 34 \rangle}$$

$$\langle 21 \rangle \langle 34 \rangle = -\langle 23 \rangle \langle 41 \rangle - \langle 24 \rangle \langle 43 \rangle$$

$$- \langle 23 \rangle \langle 41 \rangle \langle 29 \rangle [29] - \langle 24 \rangle \langle 43 \rangle \langle 29 \rangle [29] - \langle 21 \rangle \langle 34 \rangle \langle 29 \rangle [23]$$

$$\cdot \langle 13 \rangle \rightarrow \langle 21 \rangle (-\langle 12 \rangle \langle 43 \rangle - \langle 14 \rangle \langle 32 \rangle) [24] - \langle 21 \rangle \langle 23 \rangle \langle 43 \rangle [23]$$

$$\frac{\langle 21 \rangle \langle 34 \rangle}{[23] \langle 23 \rangle \langle 34 \rangle} = - \frac{\langle 23 \rangle \langle 41 \rangle - \langle 24 \rangle \langle 43 \rangle}{[23] \langle 23 \rangle \langle 34 \rangle} = - \frac{\langle 41 \rangle}{[23] \langle 34 \rangle} - \frac{\langle 24 \rangle \langle 43 \rangle}{[23] \langle 23 \rangle \langle 34 \rangle}$$

Why in the Yang-Mills Lagrangian do we have T^a gauge group generators in the fermion interactions, but not in the gauge boson kinetic term $(F_{\mu\nu})^2$?

2.23 $A_4[1234] = \epsilon_1 \cdot \epsilon_3 \cdot \epsilon_2 \cdot \epsilon_4$ from $\sqrt{2} \epsilon_1 \epsilon_2 \epsilon_3 \epsilon_4 (p_1, p_2, p_3, p_4) = \eta^{1,13} \eta^{2,24} \eta^{3,34} \eta^{4,41}$

2.24 $\epsilon_-(p, q) \cdot \epsilon_-(k, q')$
 $= \frac{\langle p | \gamma_\mu | q \rangle}{\sqrt{2} [q, p]} \cdot \frac{\langle k | \gamma_\mu | q' \rangle}{\sqrt{2} [k, q']}$

$= \frac{1}{2} \frac{1}{[q, p][k, q']} \cdot 2 \langle p k \rangle [q, q']$ by Prop 2.36

$= \frac{\langle p k \rangle [q, q']}{[q, p][k, q']}$

2.23 so for the all-minus, we have:

$\frac{\langle p_1 p_3 \rangle [q_1, q_3]}{[p_1, q_1][p_3, q_3]} \cdot \frac{\langle p_2 p_4 \rangle [q_2, q_4]}{[p_2, q_2][p_4, q_4]}$

2.25 $A_4[1^- 2^- 3^+ 4^+] = \frac{\langle 12 \rangle^4}{\langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \cdot \frac{[23]^4}{[23]^4} = \frac{(\langle 12 \rangle [23])^4 (-\langle 14 \rangle [43])^2}{\langle 23 \rangle [23]^2 [23] \langle 34 \rangle \langle 41 \rangle}$

$= \frac{[43]^2 \cdot 2 p_1 \cdot p_2}{[21]^2 \cdot 2 p_2 \cdot p_3}$

$I = \frac{1}{s} \frac{1}{t}$
 $I = \frac{1}{s} \frac{1}{t}$
 $I = \frac{1}{s} \frac{1}{t}$

(1.1)

2.27 (2.83): $A_4 [2 \bar{3} 4^+] = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$

U(1): $A_4 [1234] + A_4 [2134] + A_4 [2314] = 0$

$A_4 [1 \bar{2} 3^+ 4^+] + A_4 [2 \bar{1} 3^+ 4^+] + A_4 [2 \bar{3} 1^+ 4^+] = 0$

$$\frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} + \frac{\langle 12 \rangle^4}{\langle 21 \rangle \langle 13 \rangle \langle 34 \rangle \langle 42 \rangle} + \frac{\langle 12 \rangle^4}{\langle 23 \rangle \langle 31 \rangle \langle 41 \rangle \langle 42 \rangle} = \langle 12 \rangle^4 \left(\frac{\langle 13 \rangle \langle 42 \rangle - \langle 23 \rangle \langle 41 \rangle + \langle 12 \rangle \langle 34 \rangle}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle \langle 42 \rangle} \right)$$

= 0 by Plücker relations/Schubert (2.99) $\langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 42 \rangle + \langle 14 \rangle \langle 23 \rangle = 0$

not out of order

$A_5 [1, 4, 2, 3, 5] = - (A_5 [1, 4, 2, 3, 5] + A_5 [1, 4, 3, 2, 5] + A_5 [1, 3, 4, 2, 5])$
 $A_5 [1, 4, 2, 3, 5] = A_5 [1, 2, 3, 4, 5] - A_5 [1, 2, 4, 3, 5]$ by (2.87)

2.29 $A_5 [1, 5, 2, 3, 4] = - (A_5 [1, 2, 3, 4, 5] + A_5 [1, 2, 4, 3, 5] + A_5 [1, 4, 2, 3, 5])$

$A_5 [1, 2, 3, 5, 4] + A_5 [1, 2, 3, 4, 5] + A_5 [1, 2, 4, 3, 5] + A_5 [1, 4, 2, 3, 5] = 0$

↓ U(1)

Same

Goal: $A_5 [1, 2, 3, 4, 5] + A_5 [1, 2, 4, 3, 5] + A_5 [1, 4, 2, 3, 5] + A_5 [1, 4, 3, 2, 5] = 0$

93521

I saw that I could cancel these 2 by simply the 4 with U(1), which would also produce a term similar to

$A_5 [1, 2, 3, 5, 4] + A_5 [1, 4, 2, 3, 5] + A_5 [1, 2, 4, 3, 5] + A_5 [1, 2, 3, 4, 5] = 0$

$A_5 [1, 2, 3, 5, 4] = -A_5 [1, 4, 2, 3, 5] - A_5 [1, 2, 4, 3, 5] - A_5 [1, 2, 3, 4, 5] +$

(2.88) right U(1)!

$+ A_5 [1, 2, 3, 4, 5] + A_5 [1, 2, 4, 3, 5] + A_5 [1, 4, 2, 3, 5]$ doesn't realize the cancel

now, if I move the 2, I can both cancel AND get

121

$A_5 [1, 2, 4, 3, 5] + A_5 [1, 4, 2, 3, 5] + A_5 [1, 4, 3, 2, 5] + A_5 [1, 4, 3, 5, 2]$

= $-A_5 [1, 4, 2, 3, 5] + A_5 [1, 4, 2, 3, 5] + A_5 [1, 4, 3, 2, 5] + A_5 [1, 4, 3, 5, 2] + A_5 [1, 2, 4, 3, 5] + A_5 [1, 4, 2, 3, 5]$
 $- A_5 [1, 2, 3, 4, 5]$ by reflection

$$[EPS]A_{2,2} - [PES]A_{1,2} : (P_{1,2})$$

$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$
 $\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$
 $\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$

$$2.28 \quad A[1243] = -A[1234] - A[1324] \text{ from (2.86)}$$

$$A[1234] + A[2134] + A[2314] = 0 \text{ from (2.85)}$$

$$\Rightarrow A[1234] + A[4312] + A[4132] = 0 \text{ from reflection}$$

$$A[1234] + A[1243] + A[1324] = 0 \text{ from cyclicity}$$

$$\Rightarrow A[1243] = -A[1234] - A[1324]$$

$$(1) \cdot \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$[PS]A_{2,2} = [PES]A_{1,2}$$

$$[PS]A_{2,2} = [PES]A_{1,2}$$

$$[PS]A_{2,2} = [PES]A_{1,2}$$

$$[PS]A_{2,2} = [PES]A_{1,2} : (P_{1,2})$$

$$[PS]A_{2,2} = [PES]A_{1,2}$$

$$[PS]A_{2,2} = [PES]A_{1,2}$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$0 = [PS]A_{2,2} + [PES]A_{1,2} + 0 + 0$$

$$[PS]A_{2,2} = [PES]A_{1,2}$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$0 = [PS]A_{2,2} + [PES]A_{1,2} + 0 + 0$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

$$\frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}} = \frac{[PS]A_{2,2} - [PES]A_{1,2}}{[PS]A_{2,2} - [PES]A_{1,2}}$$

2.30

Take (2.89): $S_{14} A(234) - S_{13} A(1243)$

using (2.83) ~~is not~~ since $+++, ---, ++-,$ and $---+$ vanish, we not have MHV. So we can choose the ϕ helicity to be on: $\begin{matrix} 12 & 23 & 34 \\ 13 & 24 & 31 \\ 14 & 21 & 32 \end{matrix}$ (can consider where the symmetry eg. $A(1 \dots n) = A(2 \dots n)$ etc. can be used to simplify reduce # things to check)

$[12]: A(234) = \frac{\langle 34 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$

$S_{14} = \langle 14 \rangle [4] \text{ (angle } \cdot \frac{1}{2})$

$A(1243) = \frac{\langle 43 \rangle^4}{\langle 12 \rangle \langle 24 \rangle \langle 43 \rangle \langle 31 \rangle}$

So $S_{14} A(234) - S_{13} A(1243)$

$= \frac{\langle 34 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \cdot \langle 14 \rangle [4] - \frac{\langle 43 \rangle^4}{\langle 12 \rangle \langle 24 \rangle \langle 43 \rangle \langle 31 \rangle} \cdot \langle 13 \rangle [3] \cdot (-1)$

$S_{14} = S_{23}: \langle 14 \rangle [4] = \langle 23 \rangle [23]$
 $[4] = \frac{\langle 23 \rangle [23]}{\langle 14 \rangle}$

$p_1 + p_4 = p_2 + p_3$
 $(p_1 + p_4)^2 = (p_2 + p_3)^2$
 $p_1 \cdot p_4 = p_2 \cdot p_3$

$= - \frac{\langle 34 \rangle^4 [23]}{\langle 12 \rangle \langle 34 \rangle [4]} + \frac{\langle 34 \rangle^4 [13]}{\langle 12 \rangle \langle 24 \rangle [43]} = - \frac{\langle 34 \rangle^4}{\langle 12 \rangle \langle 34 \rangle [4]} \frac{[24] \langle 41 \rangle}{[43]} + \frac{\langle 34 \rangle^4}{\langle 12 \rangle \langle 24 \rangle [43]} \frac{[24] \langle 41 \rangle}{[43]}$

$[13]: \frac{\langle 24 \rangle^4}{S_{14} \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} - S_{13} \frac{\langle 24 \rangle^4}{\langle 12 \rangle \langle 24 \rangle \langle 43 \rangle \langle 31 \rangle}$

$= \frac{\langle 24 \rangle^4 \langle 14 \rangle [4]}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} - \frac{\langle 24 \rangle^4 \langle 13 \rangle [3]}{\langle 12 \rangle \langle 24 \rangle \langle 43 \rangle \langle 31 \rangle}$

$[13] = \frac{\langle 24 \rangle [24]}{[13]}$

$p_1 + p_3 + p_4 = 1/2 [1 + 1/2 (2+3) (3+4) [4] = 0$
 $\langle 41 \rangle$

$0 + 0 + \langle 13 \rangle [3] + \langle 14 \rangle [4] = 0 \Rightarrow \frac{\langle 41 \rangle}{[43]} = \frac{[23]}{[24]}$

$= - \frac{\langle 24 \rangle^4 [4]}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle} + \frac{\langle 24 \rangle^4 [13]}{\langle 12 \rangle \langle 24 \rangle \langle 43 \rangle}$

$= - \frac{\langle 24 \rangle^4 [4]}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle} + \frac{\langle 24 \rangle^4 \langle 24 \rangle [4]}{\langle 12 \rangle \langle 24 \rangle \langle 43 \rangle \langle 23 \rangle}$

$= - \frac{\langle 24 \rangle^4 [4]}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle} + \frac{\langle 24 \rangle^4 [4]}{\langle 12 \rangle \langle 34 \rangle \langle 23 \rangle} = 0$

$[2 \cdot 11]$
 $0 + 0 + \langle 23 \rangle [31] + \langle 24 \rangle [41] = 0$
 $\frac{[31]}{[41]} = - \frac{\langle 24 \rangle [4]}{\langle 23 \rangle}$

etc. etc.

For (2.88), (2.90) $\checkmark$ S-point will be either MHV: $\frac{\langle ij \rangle^4}{\langle 12 \rangle \dots \langle n1 \rangle}$ or MHV: $\frac{[ij]^4}{[12] \dots [n1]}$

Strong's HW 2.31

$$\text{Tr}(T_{a_1} T_{a_2} T_{a_3} T_{a_4})$$

$$\begin{matrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{matrix}$$

$$A_q^{\text{full, tree}} = \frac{h_s c_s}{s} + \frac{h_t c_t}{t} + \frac{h_u c_u}{u}$$

$$\text{Prop (2.79)} \quad A_n^{\text{full, tree}} = g^{n-2} \sum_{\text{perm } \sigma} A_n[1 \sigma(2 \dots n)] \text{Tr}(T_{a_1} T_{a_2} \dots T_{a_n})$$

$$= g^{n-2} \left(A_q[1234] \text{Tr}(T_{a_1} T_{a_2} T_{a_3} T_{a_4}) + A_q[1243] \text{Tr}(T_{a_1} T_{a_2} T_{a_4} T_{a_3}) + A_q[1324] \text{Tr}(T_{a_1} T_{a_3} T_{a_2} T_{a_4}) + A_q[1342] \text{Tr}(T_{a_1} T_{a_3} T_{a_4} T_{a_2}) + A_q[1423] \text{Tr}(T_{a_1} T_{a_4} T_{a_2} T_{a_3}) + A_q[1432] \text{Tr}(T_{a_1} T_{a_4} T_{a_3} T_{a_2}) \right)$$

1234 → 4321 by reflection
1234 → 1432 by adjoint

$$1342 \rightarrow 2431$$

$$= g^{n-2} \left(A[1234] \text{Tr}[1234] + A[1432] \text{Tr}[1234] + A[1243] \text{Tr}[1243] + A[1342] \text{Tr}[1243] + A[1324] \text{Tr}[1324] + A[1423] \text{Tr}[1324] \right) = g^{n-2} \left(\text{Tr}[1234] (A[1234] + A[1432]) + \text{Tr}[1243] (A[1243] + A[1342]) + \text{Tr}[1324] (A[1324] + A[1423]) \right)$$

$$A_q^{\text{full, tree}} = \frac{h_s c_s}{s} + \frac{h_t c_t}{t} + \frac{h_u c_u}{u}$$

$$c_s = \hat{p}_{a_1 a_2} \hat{p}_{b a_3 a_4} = \text{Tr}[(1234) - (1243) - (1342) + (1432)]$$

$$(2.68) \quad c_t = \hat{p}_{a_1 a_3} \hat{p}_{b a_2 a_4} = \text{Tr}[(1342) - (1324) - (1423) + (1243)]$$

$$c_u = \hat{p}_{a_1 a_4} \hat{p}_{b a_3 a_2} = \text{Tr}[(1423) - (1432) - (1234) + (1324)]$$

⇒ 10 $c_s + c_t + c_u = 0$ (Jacobi) holds

$$= \left[\frac{h_s}{s} (\text{Tr}(1234) - \text{Tr}(1243)) + \frac{h_t}{t} (\text{Tr}(1243) - \text{Tr}(1324)) + \frac{h_u}{u} (\text{Tr}(1324) - \text{Tr}(1234)) \right]$$

$$g^{n-2} (A[1234] + A[1432]) = 2 \left(\frac{h_s}{s} - \frac{h_u}{u} \right)$$

$$\Rightarrow u: h_s - s: h_u = \frac{g^{n-2}}{s} \cdot u \cdot s (A[1234] + A[1432])$$

$$\cancel{\frac{p_3}{q_3}} - \cancel{\frac{p_1}{q_1}} + \cancel{\frac{p_2}{q_2}} - \cancel{\frac{p_3}{q_3}} + \cancel{\frac{p_1}{q_1}} - \cancel{\frac{p_2}{q_2}} = 0$$

$$\Rightarrow A[1234] + A[1432] + A[1243] + A[1342] + A[1324] + A[1423] = 0$$

$$\bullet \text{ UCI: } A[1234] + A[2134] + A[2314] = 0 \quad \text{cyclotomic}$$

$$= A[1234] + A[1342] + A[1423] = 0$$

and

$$A[1432] + A[4132] + A[4312] = 0$$

$$A[1423] + A[1324] + A[1243] = 0$$

Zero by 2 UCI-deeply

Syna exeric: $r, s = \langle r_1 \rangle \langle r_2 \rangle$
 $p, q = \langle p_1 \rangle \langle p_2 \rangle$

$$-p_0 q_0 + p_1 q_1 + p_2 q_2 + p_3 q_3$$

$$\det \begin{pmatrix} p_0 - p_3 & q_0 - q_3 \\ -p_1 - p_2 & q_1 - q_2 \end{pmatrix} \cdot \frac{z^2}{\sqrt{p_0 - p_3} \sqrt{q_0 - q_3}} \cdot \begin{pmatrix} p_0 - p_3 & -p_1 + p_2 \\ q_0 - q_3 & -q_1 + q_2 \end{pmatrix}$$

$$\frac{[(p_0 - p_3)(q_1 - q_2) - (q_0 - q_3)(-p_1 + p_2)] [(p_0 - p_3)(-q_1 + q_2) - (q_0 - q_3)(-p_1 + p_2)]}{(p_0 - p_3)(q_0 - q_3)}$$

$$-p_0 q_1$$

$$= \left[\frac{-q_1 - q_2}{q_0 - q_3} + \frac{p_1 + p_2}{p_0 - p_3} \right] \left[\frac{-q_1 + q_2}{q_0 - q_3} + \frac{p_1 - p_2}{p_0 - p_3} \right]$$

$$= (2q_2 q_3) \frac{q_1^2 + q_2^2}{(q_0 - q_3)^2}$$

$$-q_0^2 - q_1^2 + q_2^2 + q_3^2 = 0$$

$$q_1^2 + q_2^2 = q_0^2 - q_3^2$$

$$\frac{q_0^2 - q_3^2}{(q_0 - q_3)^2} + \frac{p_0^2 - p_3^2}{(p_0 - p_3)^2} = \frac{(p_1 + p_2)(-q_1 + q_2) + (p_1 - p_2)(-q_1 - q_2)}{(p_0 - p_3)(q_0 - q_3)}$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

$$q_1^2 + q_2^2 = q_0^2 - q_3^2$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

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$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

$$[p_1] \langle p_2 \rangle = [p_2] \langle p_1 \rangle$$

2.39 Suppose $A_n [i^+, \dots, i^-, j^-, \dots, j^-, n^+] = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle} (31) = (p, q); M : \text{stair } 2 \text{ to } 0$

Suppose $k \neq i, j$ goes soft

wlog say $k=2$: $W_6: A_n \rightarrow \frac{\langle k+1, k-1 \rangle}{\langle k+1, k \rangle \langle k, k-1 \rangle} \cdot A_{n-1} [\dots, k^-, k^+, \dots]$

$$= \frac{\langle 3, 1 \rangle}{\langle 3, 2 \rangle \langle 2, 1 \rangle} \cdot A_{n-1} [1^+, 3^+, \dots, j^-, \dots, n^+]$$

can't have $k=i, j$ or else A_{n-1} would go to zero

$$\frac{\langle 31 \rangle \langle 1j \rangle^4}{\langle 32 \rangle \langle 21 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle} [i^+]$$

$$= \frac{\langle 1j \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \dots \langle n1 \rangle} [i^+]$$

minus sign from $\frac{\langle 31 \rangle}{\langle 13 \rangle}$?

$$(2.126) \frac{\langle 12 \rangle^3}{\langle 23 \rangle \dots \langle n1 \rangle}$$

$$\xrightarrow[p_1 = z p]{p_k = (1-z)p}$$

$$\frac{\langle 12 \rangle^3}{z^3 \langle p2 \rangle^3} \xrightarrow{p_1 = z p} \frac{\langle 12 \rangle^3}{z^3 \langle (1-z)p, z p \rangle^3}$$

$$\frac{\langle n+1, n \rangle}{\langle n+1, (1-z)p \rangle + \langle (1-z)p, z p \rangle} = \frac{\langle n+1, n \rangle}{\langle n+1, p \rangle}$$

$$\frac{(\sqrt{z})^3 \langle p2 \rangle^3}{\langle 23 \rangle \dots \langle n+1, p \rangle \cdot \sqrt{z} \langle n1 \rangle}$$

$$= \frac{z^2}{\sqrt{z(1-z)} \langle n1 \rangle} \cdot \frac{\langle p2 \rangle^3}{\langle 23 \rangle \dots \langle n+1, p \rangle}$$

$$\frac{\langle n+1, n \rangle}{\langle n+1, p \rangle \langle p, n \rangle} = \frac{\langle n+1, n \rangle}{\langle n+1, p \rangle \langle p, n \rangle}$$

$$(2.40) \Delta_n [1^- 2^- 3^+ 4^+ \dots n^+]$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \xrightarrow{3114} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

$$\langle 12 \rangle \langle 2p \rangle \sqrt{1-z} \langle 34 \rangle \sqrt{z} \langle ps \rangle \dots \langle nd \rangle$$

$$= \frac{1}{\sqrt{z(z-1)} \langle 34 \rangle} \frac{\langle 12 \rangle^3}{\langle 12 \rangle \langle 2p \rangle \langle ps \rangle \dots \langle n1 \rangle}$$

splitting function

we could have $i-j$ be non adjacent

$$\text{Split } (n^+, 1^-) = \frac{z^2}{\sqrt{z(1-z)} \langle n1 \rangle}$$

is consistent with little group scaling

$$(2.49): \langle n1 \rangle^{h_p - h_n - h_1}$$

$$\uparrow$$

$$= -1$$

$$\text{so } h_p - h_n - h_1 = -1$$

$$\text{so } h_p = -1$$

$$|1\rangle = \sqrt{z} |p\rangle$$

$$(t|1\rangle) = \sqrt{z} |p\rangle$$

$$\downarrow$$

$$t^{-2h_1} A_n$$

$$t^{2h_1} A_n$$

$$\frac{\sqrt{z} t^2}{\sqrt{z(1-z)}} \rightarrow \frac{t^2 z^2}{\sqrt{t z - t^2 z^2}} = \frac{t^2 z^2}{\sqrt{t z} \sqrt{1-tz}} = \frac{(tz)^{3/2}}{\sqrt{1-tz}}$$

$$A_n = \sum_{\substack{\text{poles} \\ z=0}} \frac{\text{Res}}{z} \hat{A}_n(z) + \sum_{\substack{\text{poles} \\ z=\infty}} \frac{\text{Res}}{z} \hat{A}_n(z)$$

$$\hat{A}_n(z) = \hat{A}_n(z)$$

$$\text{so that for } z \rightarrow \infty$$

$$C_n = \text{Res}_{z=0} \hat{A}_n(z) = 0 \Rightarrow \text{Res}_{z=0} \frac{1}{z} \hat{A}_n(z) = 0$$

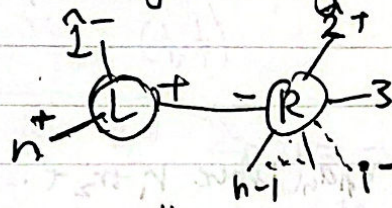
Sony, Chapter 3 was the first chapter in Scotland's Amplitudes ~~and~~
 in Gauge Theory and Gravity that I read back in 2019 - at the time, I
 sort of tried the exercises while working out some of the details in the text,
 so I'm having trouble separating out where the exercises start/end, and so I've just
 decided to scan the entire section in my notebook - apologies for the messiness.

JP
 8/23/2026

p57 Bluey Hany

Exercise: MHV for nonadjacent: say: $[1^-, 2^+, \dots, i-1^+, i^-, i+1^+, \dots, n^+]$

$C, + \rightarrow$ BCFW shift



is really in that doesn't vary

$$\frac{[n \hat{P}_{n1}]^3}{[\hat{P}_{n1} \uparrow][\uparrow n]} \cdot \frac{1}{[n] \langle 1n \rangle} \cdot \frac{\langle 1 \hat{P}_{n1} \rangle^4}{\langle \hat{P}_{n1} 2 \rangle \langle 2 3 \rangle \dots \langle n-1 \hat{P}_{n1} \rangle}$$

$$- [n \hat{P}_{n1}] \langle \hat{P}_{n1} i \rangle = [n | \hat{p}_i + p_n | i \rangle = [n | p_i | i \rangle = [n \uparrow] \langle \uparrow i \rangle$$

$$[n \uparrow] = [n (1 + z 2)] = ([n 1] + z [n 2]) \langle 1 i \rangle = [n 1] \langle 1 i \rangle + z [n 2] \langle 1 i \rangle$$

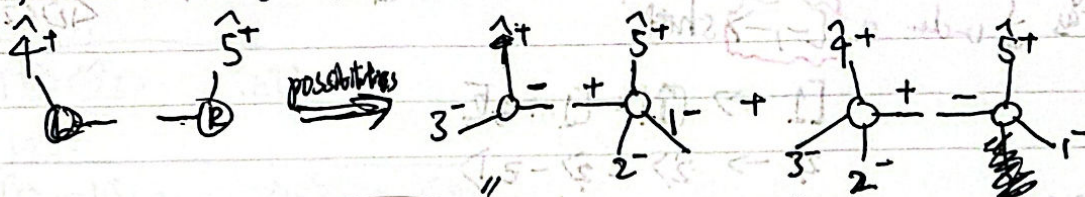
don't get $\langle 1 2 \rangle = \langle 1 2 \rangle$ as in
(3.22), (3.23) p56 BH

$$\begin{aligned} - \langle 2 \hat{P}_{n1} \rangle [\hat{P}_{n1} \uparrow] &= - \langle 2 | \hat{p}_{n1} | \uparrow \rangle \\ &= - \langle 2 | n + \uparrow | \uparrow \rangle \\ &= - \langle 2 | n | \uparrow \rangle \\ &= - \langle 2 n \rangle [n \uparrow] \\ &= - (\langle 2 n \rangle - z \langle 1 n \rangle) ([n 1] + z [n 2]) \\ &= - (\langle 2 n \rangle [n 1] + z \langle 2 n \rangle [n 2] - z \langle 1 n \rangle [n 1] - z^2 \langle 1 n \rangle [n 2]) \\ &= - \langle 2 | p_n | \uparrow \rangle + z (2 p_2 \cdot p_n - 2 p_1 \cdot p_n) - z^2 \langle 1 | p_n | 2 \rangle \end{aligned}$$

may
go to p202

PS7 Biny Khay

Exercise: $[+7]$ sharp on A_5 $[123^4 4^+ 5^+]$



$$\frac{\langle 3 \hat{p}_{34} \rangle^3}{\langle 3 \hat{q} \rangle \langle 1 \hat{q} \rangle} \cdot \frac{1}{p_{34}} \cdot \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 2 \hat{p}_{34} \rangle \langle \hat{p}_{34} 5 \rangle \langle 5 \rangle}$$

$$+ \frac{\langle 23 \rangle^4}{\langle 23 \rangle \langle 3 \hat{q} \rangle \langle \hat{q} \hat{p}_{15} \rangle \langle \hat{p}_{15} 2 \rangle} \cdot \frac{1}{p_{15}} \cdot \frac{\langle 1 \hat{p}_{15} \rangle^3}{\langle \hat{p}_{15} \hat{s} \rangle \langle 1 \hat{s} \rangle}$$

Or dual case: $\hat{p}_{15}^2 = 2p_1 \cdot p_5 = \langle 1 \hat{s} \rangle \langle 1 \hat{s} \rangle$
 $= \langle 1 \hat{s} \rangle \langle 1 \hat{s} \rangle$

For given moment, the only way for p_{15} to vanish is for z to take a value st. $\langle 1 \hat{s} \rangle = 0$

$$|\hat{p}_{15}\rangle \langle \hat{p}_{15}| = \hat{p}_{15} |1\rangle = (p_1 + p_5) |1\rangle = \hat{p}_5 |1\rangle = \hat{s} |1\rangle \langle 1| = 0 \text{ so } \langle \hat{p}_{15} | = 0$$

$$|\hat{p}_{15}\rangle \langle \hat{p}_{15} \hat{s}| = (p_1 + p_5) |3\rangle = 1 \langle 1 \hat{s} \rangle = 0 \Rightarrow \langle \hat{p}_{15} \hat{s} | = 0$$

$$\frac{0^3}{0^2} = 0$$

Thus we only have the left diagram

$$\langle 3 \hat{p}_{34} \rangle \Rightarrow \frac{\langle 3 | \hat{p}_{34} \rangle \langle \hat{p}_{34} |}{\langle 3 \hat{q} \rangle \langle 3 \hat{q} \rangle}$$

ps4 Elong theory

Carmer yourself that for large z the amplitude (3.1) $A_n[1^- 2^- 3^+ \dots n^+] = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$ falls off as $\frac{1}{z}$ under a $[-, -]$ shift

$$|1\rangle \rightarrow |1\rangle = |1\rangle + z|2\rangle$$

$$|2\rangle \rightarrow |2\rangle = |2\rangle - z|1\rangle$$

$$A_n[1^- 2^- 3^+ \dots n^+] = \frac{(\langle 1(2-z|1)\rangle)^4}{\langle 1(2-z|1)\rangle \langle 23 \rangle \dots \langle n1 \rangle} = \frac{\langle 127-z\langle 11 \rangle^4}{\langle 127-z\langle 11 \rangle \langle 23 \rangle - z\langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle - z\langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle} = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \dots \langle n1 \rangle - z\langle 12 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle}$$

$$\frac{A}{B-Cx} \approx \frac{A}{C} \frac{1}{1-\frac{B}{C}x} = \frac{A}{C}$$

$$\rightarrow \frac{A}{B} \frac{1}{1-\frac{B}{C}x} = \frac{A}{B} (1 + \frac{B}{C}x + \frac{B^2}{C^2}x^2 + \dots)$$

the z term dominates as $z \rightarrow \infty$

$$= \frac{\langle 12 \rangle^4}{z\langle 12 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle} = \frac{1}{z} \frac{\langle 23 \rangle}{\langle 13 \rangle} \cdot \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

case 1

$$[-, +]$$

$$|1\rangle \rightarrow |1\rangle + z|3\rangle$$

$$|3\rangle \rightarrow |3\rangle - z|1\rangle$$

$$A_n[1^- 2^- 3^+ \dots n^+]$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 2(3-z|1)\rangle \langle 3+z\langle 11 \rangle \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle (\langle 23 \rangle - z\langle 21 \rangle) (\langle 34 \rangle + z\langle 14 \rangle) \langle 45 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle (\langle 23 \rangle \langle 34 \rangle + z(\langle 14 \rangle \langle 23 \rangle - \langle 21 \rangle \langle 34 \rangle) - z^2 \langle 21 \rangle \langle 14 \rangle) \langle 45 \rangle \dots \langle n1 \rangle}$$

$$\rightarrow \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 21 \rangle \langle 14 \rangle \langle 45 \rangle \dots \langle n1 \rangle} \frac{1}{z} \text{ as } z \rightarrow \infty$$

$$b. A_n \left[\boxed{1^+} \boxed{2^+} 3^- 4^+ \dots n^+ \right]$$

case 2

$$\begin{aligned}
 &= \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \\
 &= \frac{\langle 13 \rangle^4}{\langle 1(2-z1) \rangle \langle 2-z(1)3 \rangle \langle 34 \rangle \dots \langle n1 \rangle} \\
 &= \frac{\langle 13 \rangle^4}{(\langle 12 \rangle - z\langle 1 \rangle) (\langle 23 \rangle - z\langle 13 \rangle) \langle 34 \rangle \dots \langle n1 \rangle} \\
 &= \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \dots \langle n1 \rangle - z \cdot \langle 12 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle} \rightarrow \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle} \cdot \frac{1}{z} \langle 12 \rangle \langle 34 \rangle \dots
 \end{aligned}$$

$[+ \rightarrow]$

case 1

$$A_n \left[\boxed{1^-} 2^- \boxed{3^+} \dots n^+ \right]$$

$$\begin{aligned}
 &[3 \rightarrow [3+z]] \quad 1 \rightarrow [1-z] \\
 &= \frac{(\langle 1-z3 \rangle)^4}{(\langle 1-z3 \rangle \langle 23 \rangle \dots \langle n(1-z3) \rangle)} \\
 &= \frac{(\langle 12 \rangle - z\langle 32 \rangle)^4}{(\langle n1 \rangle - z\langle n3 \rangle) (\langle 42 \rangle - z\langle 32 \rangle) \langle 23 \rangle \dots \langle n1 \rangle} \\
 &\rightarrow \frac{(\langle 12 \rangle + z\langle 23 \rangle)^4}{(\langle 12 \rangle \langle n1 \rangle + z(\langle 3n \rangle \langle 12 \rangle + \langle 23 \rangle \langle n1 \rangle) + z^2 \langle n3 \rangle \langle 32 \rangle \langle 23 \rangle \dots \langle n1 \rangle)} \rightarrow \frac{z^2 \langle 23 \rangle^4}{z^2 \langle n3 \rangle \langle 32 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \cdot \frac{1}{z^2}
 \end{aligned}$$

case 2

$$A_n \left[\boxed{1^-} 2^- 3^+ \dots \boxed{n^+} \right]$$

$$\begin{aligned}
 &[n \rightarrow [n+z]] \quad 4 \rightarrow [4-z] \\
 &= \frac{(\langle 1-zn \rangle)^4}{(\langle 1-zn \rangle \langle 23 \rangle \dots \langle n(1-zn) \rangle)} \\
 &= \frac{(\langle 12 \rangle - z\langle n2 \rangle)^4}{(\langle 42 \rangle - z\langle n2 \rangle) \langle 23 \rangle \dots \langle n(1-zn) \rangle} \\
 &\rightarrow \frac{z^4 \langle n2 \rangle^4}{-z \langle n2 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \rightarrow \frac{z^3 \langle n2 \rangle^4}{\langle n2 \rangle \langle 23 \rangle \dots \langle n1 \rangle}
 \end{aligned}$$

seems like $\frac{1}{z}$ for non-adjacent

think of n like x : n is a label for the basis, just like x in $|x\rangle$ is a label for basis of an Hilbert space consists of L^2 -functions on $\mathbb{R}$
(think of n_1, n_2 like $x \cdot y$ or $x \otimes y$?)

$[++]$

adjoint

$$A_n [1^- 2^- 3^+ 4^+ 5^+ \dots n^+]$$

$$[3 \rightarrow [3 + z[4, \langle 4 \rightarrow \langle 4 - z[3$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

$$\langle 12 \rangle \langle 23 \rangle (\langle 34 \rangle - z \langle 35 \rangle) (\langle 45 \rangle - z \langle 35 \rangle) \dots \langle n1 \rangle$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle = z \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 35 \rangle \dots \langle n1 \rangle$$

$$\rightarrow \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 35 \rangle \dots \langle n1 \rangle} \boxed{\frac{1}{z}} \text{ as } z \rightarrow \infty$$

kinadjoint $A_n [1^- 2^- 3^+ 4^+ 5^+ \dots n^+]$

$$[3 \rightarrow [3 + z[5, \langle 5 \rightarrow \langle 5 - z[3$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

$$\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle (\langle 45 \rangle - z \langle 43 \rangle) (\langle 56 \rangle - z \langle 36 \rangle) \dots \langle n1 \rangle$$

$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

$$[\langle 45 \rangle \langle 56 \rangle - z(\langle 43 \rangle \langle 56 \rangle + \langle 45 \rangle \langle 36 \rangle) + z^2 \langle 43 \rangle \langle 36 \rangle] \cdot \langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle$$

$$\rightarrow \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle} \boxed{\frac{1}{z^2}} \text{ as } z \rightarrow \infty$$

Now do $\boxed{[- \rightarrow \text{has adjoint}]}$

$$A_n [1^- 2^+ 3^+ 4^+ \dots n^+]$$

$$[1 \rightarrow 0 + z [3$$

$$[3 \rightarrow [3 - z [1$$

$$= \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 1(3-z1) \rangle^4}{\langle 12 \rangle \langle 2(3-z1) \rangle \langle 3-z1 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 2(3-z1) \rangle \langle 3-z1 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 2(3-z1) \rangle \langle 3-z1 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 2(3-z1) \rangle \langle 3-z1 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 2(3-z1) \rangle \langle 3-z1 \rangle \dots \langle n1 \rangle}$$

$$= \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 2(3-z1) \rangle \langle 3-z1 \rangle \dots \langle n1 \rangle}$$

$$\rightarrow \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 2(3-z1) \rangle \langle 3-z1 \rangle \dots \langle n1 \rangle} \text{ as } z \rightarrow \infty$$

$$\rightarrow \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 2(3-z1) \rangle \langle 3-z1 \rangle \dots \langle n1 \rangle} \text{ as } z \rightarrow \infty$$

Ex 3.3 p 54

Adm of a $[1, 2]$ shift: $\langle 12 \rangle^4 / \langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle - z \langle 12 \rangle \langle 34 \rangle \dots \langle n1 \rangle$

from 2 pages ago

$$\text{Simple pole: } z = \frac{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \dots \langle n1 \rangle}{\langle 12 \rangle \langle 34 \rangle \dots \langle n1 \rangle} = \frac{\langle 23 \rangle}{\langle 13 \rangle}$$

Calculate the residue of $\frac{\hat{A}_n(z)}{z}$ at this pole

$$\text{For simple pole at } z=a, \text{ Res } f(z) = \lim_{z \rightarrow a} (z-a) f(z) = \lim_{z \rightarrow \frac{\langle 23 \rangle}{\langle 13 \rangle}} \left(z - \frac{\langle 23 \rangle}{\langle 13 \rangle} \right) \cdot \frac{\hat{A}_n(z)}{z}$$

$$= \lim_{z \rightarrow \frac{\langle 23 \rangle}{\langle 13 \rangle}} \frac{z \langle 12 \rangle^4 - \langle 23 \rangle \langle 12 \rangle^4}{\langle 13 \rangle} \cdot \frac{\hat{A}_n(z)}{z} = \lim_{z \rightarrow \frac{\langle 23 \rangle}{\langle 13 \rangle}} \frac{z \langle 12 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle - \langle 12 \rangle \langle 23 \rangle \langle n1 \rangle}{\langle 12 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle} \cdot \frac{1}{z} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle n1 \rangle}$$

$$= \lim_{z \rightarrow \frac{\langle 23 \rangle}{\langle 13 \rangle}} \frac{1}{z} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle} = - \frac{1}{\langle 23 \rangle \langle 13 \rangle} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 13 \rangle \langle 34 \rangle \dots \langle n1 \rangle}$$

$$= - \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \dots \langle n1 \rangle}$$

← this is just negative of Parke-Taylor
Compare with (3.4): $A_n = - \sum_{z \neq z_i} \text{Res } \frac{\hat{A}_n(z)}{z} \cdot B_n$
 $z_i \neq 0$

$$A_n = - \text{Res}_{z=\frac{\langle 23 \rangle}{\langle 13 \rangle}} \left(- \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \right) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

Since we only have one pole, so one non zero residue

(Ex 3.3 p 54 cont.)

Try to repeat for a $|1, 3\rangle$ state

a) $E_1 = -7 = |1, 3\rangle: A_n = \frac{\langle 1, 3 | \psi \rangle}{\langle 1, 3 | \psi \rangle} = \frac{\langle 1, 3 | \psi \rangle}{\langle 1, 3 | \psi \rangle} = \dots \langle n | \psi \rangle$

b) $E_1 = -7 = |1, 3\rangle: A_n = \frac{\langle 1, 3 | \psi \rangle}{\langle 1, 3 | \psi \rangle} = \frac{\langle 1, 3 | \psi \rangle}{\langle 1, 3 | \psi \rangle} = \dots \langle n | \psi \rangle$

There both have double poles

a) $\langle 2, 3 | \psi \rangle - z \langle 2, 1 | \psi \rangle + \langle 2, 3 | \psi \rangle + z^2 \langle 2, 1 | \psi \rangle = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\langle 2, 1 | \psi \rangle \pm \sqrt{\langle 2, 1 | \psi \rangle^2 - 4 \langle 2, 1 | \psi \rangle \langle 2, 3 | \psi \rangle}}{2 \langle 2, 3 | \psi \rangle}$$

$$= \frac{1}{2} \left(\frac{\langle 2, 1 | \psi \rangle}{\langle 2, 3 | \psi \rangle} + \frac{\langle 1, 4 | \psi \rangle}{\langle 3, 4 | \psi \rangle} \right) \pm \frac{1}{2 \langle 2, 3 | \psi \rangle} \sqrt{\langle 2, 1 | \psi \rangle^2 + \langle 2, 3 | \psi \rangle^2 + 2 \langle 2, 1 | \psi \rangle \langle 2, 3 | \psi \rangle - 4 \langle 2, 1 | \psi \rangle \langle 2, 3 | \psi \rangle}$$

$$= \frac{1}{2} \left(\frac{\langle 2, 1 | \psi \rangle}{\langle 2, 3 | \psi \rangle} + \frac{\langle 1, 4 | \psi \rangle}{\langle 3, 4 | \psi \rangle} \right) \pm \frac{1}{2} \left(\frac{\langle 2, 1 | \psi \rangle}{\langle 2, 3 | \psi \rangle} - \frac{\langle 1, 4 | \psi \rangle}{\langle 3, 4 | \psi \rangle} \right)$$

$$= \frac{\langle 2, 1 | \psi \rangle}{\langle 2, 3 | \psi \rangle} \text{ or } \frac{\langle 1, 4 | \psi \rangle}{\langle 3, 4 | \psi \rangle}$$

Why 3.4: $A_n = \text{Res}_{z=0} \frac{A_n(z)}{z} = - \text{Res}_{z=\frac{\langle 2, 1 | \psi \rangle}{\langle 2, 3 | \psi \rangle}} \frac{A_n(z)}{z} - \text{Res}_{z=\frac{\langle 1, 4 | \psi \rangle}{\langle 3, 4 | \psi \rangle}} \frac{A_n(z)}{z}$

$$= - \lim_{z \rightarrow a} \left((z-a) \frac{\langle 1, 2 | \psi \rangle}{(z-a)(z-b) \langle 1, 2 | \psi \rangle \dots \langle n, 1 | \psi \rangle} \right) - \lim_{z \rightarrow b} (z-b) \frac{\langle 1, 2 | \psi \rangle}{\langle n, 2 | \psi \rangle \dots \langle n, 1 | \psi \rangle (z-a)(z-b)}$$

$$= - \frac{\langle 1, 2 | \psi \rangle}{\left(\frac{\langle 2, 1 | \psi \rangle}{\langle 2, 3 | \psi \rangle} - \frac{\langle 1, 4 | \psi \rangle}{\langle 3, 4 | \psi \rangle} \right) \langle 1, 2 | \psi \rangle \dots \langle n, 1 | \psi \rangle} = 0?$$

What's $\left(z - \frac{\langle 2, 1 | \psi \rangle}{\langle 2, 3 | \psi \rangle} \right) \left(\frac{\langle 1, 2 | \psi \rangle}{\langle 2, 3 | \psi \rangle \langle 3, 4 | \psi \rangle - z (\langle 2, 1 | \psi \rangle \langle 3, 4 | \psi \rangle + \langle 2, 3 | \psi \rangle \langle 1, 4 | \psi \rangle) + z^2 \langle 2, 3 | \psi \rangle \langle 1, 4 | \psi \rangle} \right)$

$$\frac{1}{z\langle 21 \rangle \langle 14 \rangle}$$

$$\begin{aligned} & z\langle 21 \rangle \langle 14 \rangle - z\langle 21 \rangle \langle 34 \rangle + \langle 23 \rangle \langle 14 \rangle + \langle 23 \rangle \langle 34 \rangle \left| z - \frac{\langle 21 \rangle}{\langle 23 \rangle} \right. \\ & \left. z - \left(\frac{\langle 14 \rangle}{\langle 34 \rangle} + \frac{\langle 23 \rangle}{\langle 21 \rangle} \right) + \frac{1}{z} \frac{\langle 23 \rangle \langle 34 \rangle}{\langle 21 \rangle \langle 14 \rangle} \right. \\ & \left(z - \frac{\langle 21 \rangle}{\langle 23 \rangle} \right) \left(z - \frac{\langle 14 \rangle}{\langle 34 \rangle} \right) = z^2 - z \left(\frac{\langle 21 \rangle}{\langle 23 \rangle} + \frac{\langle 14 \rangle}{\langle 34 \rangle} \right) + \frac{\langle 21 \rangle \langle 14 \rangle}{\langle 23 \rangle \langle 34 \rangle} \\ & = \frac{z^2 \langle 23 \rangle \langle 34 \rangle - z \langle 21 \rangle \langle 34 \rangle - z \langle 14 \rangle \langle 23 \rangle + \langle 21 \rangle \langle 14 \rangle}{\langle 23 \rangle \langle 34 \rangle} \end{aligned}$$

or

$$\begin{aligned} & \left(z - \frac{\langle 23 \rangle}{\langle 21 \rangle} \right) \left(z - \frac{\langle 34 \rangle}{\langle 14 \rangle} \right) = z^2 - z \left(\frac{\langle 23 \rangle}{\langle 21 \rangle} + \frac{\langle 34 \rangle}{\langle 14 \rangle} \right) + \frac{\langle 23 \rangle \langle 34 \rangle}{\langle 21 \rangle \langle 14 \rangle} \\ & = \frac{z^2 \langle 21 \rangle \langle 14 \rangle - z \langle 23 \rangle \langle 14 \rangle + \langle 34 \rangle \langle 21 \rangle + \langle 23 \rangle \langle 34 \rangle}{\langle 21 \rangle \langle 14 \rangle} \end{aligned}$$

$$\left(z - \frac{\langle 21 \rangle}{\langle 23 \rangle} \right) \left(z - \frac{\langle 34 \rangle}{\langle 14 \rangle} \right) = \frac{z^2 \langle 23 \rangle \langle 14 \rangle - z \langle 21 \rangle \langle 14 \rangle + \langle 34 \rangle \langle 23 \rangle + \langle 21 \rangle \langle 34 \rangle}{\langle 23 \rangle \langle 14 \rangle}$$

$$\langle 21 \rangle \langle 14 \rangle \cdot \left(z - \frac{\langle 23 \rangle}{\langle 21 \rangle} \right) \left(z - \frac{\langle 34 \rangle}{\langle 14 \rangle} \right) = z^2 - z \left(\frac{\langle 23 \rangle}{\langle 21 \rangle} + \frac{\langle 34 \rangle}{\langle 14 \rangle} \right) + \frac{\langle 23 \rangle \langle 34 \rangle}{\langle 21 \rangle \langle 14 \rangle}$$

$$\begin{aligned} \text{Thus res}_{z=a} = \text{res}_{z=b} &= \frac{1}{z} \frac{\langle 12 \rangle^4}{\langle 23 \rangle \langle 21 \rangle \langle 14 \rangle} = -\text{res}_{z \rightarrow \frac{\langle 23 \rangle}{\langle 21 \rangle}} \frac{1}{z} \frac{\langle 12 \rangle^4}{\langle 23 \rangle \langle 21 \rangle \langle 14 \rangle} \left(z - \frac{\langle 23 \rangle}{\langle 21 \rangle} \right) \left(z - \frac{\langle 34 \rangle}{\langle 14 \rangle} \right) \langle 12 \rangle \langle 45 \rangle \dots \langle n1 \rangle \cdot \left(z - \frac{\langle 23 \rangle}{\langle 21 \rangle} \right) \\ &= -\text{res}_{z \rightarrow \frac{\langle 34 \rangle}{\langle 14 \rangle}} \frac{1}{z} \frac{\langle 12 \rangle^4}{\langle 23 \rangle \langle 21 \rangle \langle 14 \rangle} \left(z - \frac{\langle 23 \rangle}{\langle 21 \rangle} \right) \left(z - \frac{\langle 34 \rangle}{\langle 14 \rangle} \right) \langle 12 \rangle \langle 45 \rangle \dots \langle n1 \rangle \cdot \left(z - \frac{\langle 34 \rangle}{\langle 14 \rangle} \right) \end{aligned}$$

$$= \frac{1}{\langle 23 \rangle \langle 21 \rangle \langle 14 \rangle} \frac{\langle 12 \rangle^4}{\langle 21 \rangle \langle 45 \rangle} \langle 12 \rangle \langle 45 \rangle \dots \langle n1 \rangle - \frac{1}{\langle 23 \rangle \langle 21 \rangle \langle 14 \rangle} \frac{\langle 12 \rangle^4}{\langle 14 \rangle \langle 45 \rangle} \langle 12 \rangle \langle 45 \rangle \dots \langle n1 \rangle$$

$$= -\frac{\langle 12 \rangle^4}{\langle 12 \rangle^2 \langle 14 \rangle^2 \langle 23 \rangle \langle 21 \rangle \langle 14 \rangle} \langle 45 \rangle \dots \langle n1 \rangle - \frac{\langle 12 \rangle^4}{\langle 12 \rangle^2 \langle 14 \rangle^2 \langle 23 \rangle \langle 21 \rangle \langle 14 \rangle} \langle 45 \rangle \dots \langle n1 \rangle$$

$$\begin{aligned} &= \frac{\langle 12 \rangle^4}{\langle 12 \rangle^2 \langle 14 \rangle^2 \langle 45 \rangle \dots \langle n1 \rangle} \left(1 - \frac{\langle 34 \rangle \langle 13 \rangle^2}{\langle 45 \rangle \langle 23 \rangle} \right) \rightarrow \frac{\langle 14 \rangle \langle 21 \rangle - \langle 34 \rangle \langle 23 \rangle}{\langle 14 \rangle \langle 21 \rangle} \frac{1}{\langle 23 \rangle \langle 14 \rangle} \frac{\langle 12 \rangle^2 \langle 23 \rangle \langle 34 \rangle}{\langle 21 \rangle^2 \langle 23 \rangle \langle 14 \rangle} - \frac{\langle 34 \rangle^2 \langle 23 \rangle \langle 34 \rangle \langle 23 \rangle}{\langle 21 \rangle^2 \langle 14 \rangle^2 \langle 23 \rangle \langle 14 \rangle} \\ &= \frac{\langle 12 \rangle^4}{\langle 45 \rangle \dots \langle n1 \rangle} \left(\frac{\langle 14 \rangle \langle 21 \rangle - \langle 34 \rangle \langle 23 \rangle}{\langle 23 \rangle \langle 14 \rangle^2 - \langle 21 \rangle \langle 14 \rangle \langle 23 \rangle \langle 34 \rangle} - \frac{1}{\langle 34 \rangle^2 \langle 21 \rangle^2 - \langle 14 \rangle \langle 21 \rangle \langle 34 \rangle \langle 23 \rangle} \right) \end{aligned}$$

$$\frac{(\langle 34 \rangle^2 \langle 21 \rangle^2 - \langle 14 \rangle \langle 21 \rangle \langle 34 \rangle \langle 23 \rangle) - (\langle 23 \rangle^2 \langle 14 \rangle^2 - \langle 14 \rangle \langle 21 \rangle \langle 34 \rangle \langle 23 \rangle)}{(\langle 34 \rangle^2 \langle 21 \rangle^2 - \langle 14 \rangle \langle 21 \rangle \langle 34 \rangle \langle 23 \rangle) (\langle 23 \rangle^2 \langle 14 \rangle^2 - \langle 14 \rangle \langle 21 \rangle \langle 34 \rangle \langle 23 \rangle)}$$

$$= \frac{\langle 34 \rangle^2 \langle 21 \rangle^2 - \langle 23 \rangle^2 \langle 14 \rangle^2}{2 \langle 34 \rangle \langle 21 \rangle^2 \langle 23 \rangle \langle 14 \rangle^2 - \langle 23 \rangle^3 \langle 14 \rangle^3 \langle 21 \rangle \langle 34 \rangle - \langle 21 \rangle^3 \langle 34 \rangle^2 \langle 23 \rangle \langle 14 \rangle}$$

$$\text{WB: } = \frac{1}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle}$$

$$= \frac{\langle 34 \rangle^2 \langle 21 \rangle^2 - \langle 23 \rangle^2 \langle 14 \rangle^2}{\langle 23 \rangle^2 \langle 14 \rangle^2 (\langle 34 \rangle \langle 21 \rangle^2 - \langle 23 \rangle \langle 14 \rangle \langle 21 \rangle \langle 34 \rangle) + \langle 21 \rangle^2 \langle 34 \rangle^2 (\langle 23 \rangle \langle 14 \rangle^2 - \langle 23 \rangle \langle 14 \rangle \langle 21 \rangle \langle 34 \rangle)}$$

$$\frac{\langle 12 \rangle^3 \langle 34 \rangle^3 \langle 23 \rangle - \langle 23 \rangle^3 \langle 12 \rangle \langle 34 \rangle \langle 14 \rangle^2}{2 \langle 34 \rangle \langle 21 \rangle^2 \langle 23 \rangle \langle 14 \rangle^2 - \langle 23 \rangle^3 \langle 14 \rangle^3 \langle 21 \rangle \langle 34 \rangle - \langle 21 \rangle^3 \langle 34 \rangle^2 \langle 23 \rangle \langle 14 \rangle} \stackrel{?}{=} 1$$

seems to be complicated, look at first few terms:

$$\frac{1}{2} \frac{\langle 12 \rangle^4}{\langle 21 \rangle \langle 14 \rangle (\frac{\langle 23 \rangle}{\langle 21 \rangle} - \frac{\langle 34 \rangle}{\langle 14 \rangle}) \langle 12 \rangle \langle 14 \rangle \dots \langle n1 \rangle} - \frac{1}{2} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 14 \rangle (\frac{\langle 23 \rangle}{\langle 21 \rangle} - \frac{\langle 34 \rangle}{\langle 14 \rangle}) \langle 12 \rangle \langle 14 \rangle \dots \langle n1 \rangle}$$

$$= - \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 21 \rangle \langle 14 \rangle \langle 14 \rangle \dots \langle n1 \rangle (\frac{\langle 23 \rangle}{\langle 21 \rangle} - \frac{\langle 34 \rangle}{\langle 14 \rangle}) (\frac{1}{2} + \frac{1}{2})}$$

$$= - \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 21 \rangle \langle 14 \rangle \langle 14 \rangle \dots \langle n1 \rangle (\frac{\langle 23 \rangle}{\langle 21 \rangle} - \frac{\langle 34 \rangle}{\langle 14 \rangle}) (\frac{\langle 23 \rangle}{\langle 21 \rangle} + \frac{\langle 34 \rangle}{\langle 14 \rangle})}$$

$$= - \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 21 \rangle \langle 14 \rangle \langle 14 \rangle \dots \langle n1 \rangle (\frac{\langle 23 \rangle^2}{\langle 21 \rangle^2} - \frac{\langle 34 \rangle^2}{\langle 14 \rangle^2})}$$

$$= - \frac{\langle 12 \rangle^4 \langle 21 \rangle^2 \langle 14 \rangle^2}{\langle 12 \rangle \langle 21 \rangle \langle 14 \rangle \langle 14 \rangle \dots \langle n1 \rangle (\langle 14 \rangle^2 \langle 23 \rangle^2 - \langle 21 \rangle^2 \langle 34 \rangle^2)}$$

$$= \frac{\langle 12 \rangle^4 \langle 14 \rangle}{\langle 14 \rangle \dots \langle n1 \rangle (\langle 14 \rangle^2 \langle 23 \rangle^2 - \langle 21 \rangle^2 \langle 34 \rangle^2)} \quad \text{WB: } = \frac{1}{\langle 14 \rangle \langle 23 \rangle^2 \langle 21 \rangle^2 \langle 34 \rangle^2} = \frac{1}{\langle 14 \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle}$$

$$\text{So WB: } \langle 14 \rangle^2 \langle 23 \rangle^2 - \langle 21 \rangle^2 \langle 34 \rangle^2 = \langle 14 \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle$$

$$\begin{aligned} &\stackrel{\text{Schock}}{=} \langle 14 \rangle \langle 23 \rangle (\langle 13 \rangle \langle 24 \rangle - \langle 23 \rangle \langle 14 \rangle) \\ &= \langle 14 \rangle \langle 23 \rangle \langle 13 \rangle \langle 24 \rangle - \langle 23 \rangle^2 \langle 14 \rangle^2 \\ &= \langle 14 \rangle \langle 23 \rangle (\langle 23 \rangle \langle 14 \rangle^2 - \langle 13 \rangle \langle 14 \rangle \langle 23 \rangle \langle 14 \rangle) + \langle 23 \rangle^2 \langle 14 \rangle^2 \\ &= \langle 14 \rangle \langle 23 \rangle^2 - \langle 14 \rangle^2 \end{aligned}$$

WB: $v_{15} + v_{16} = \text{Pohle-Tyler}$
 $v_{15} = a, v_{16} = b$

FAIL
182

$$D) A_h = \langle 12 \rangle^4$$

$$(\frac{1}{2}^2 \langle 21 \rangle \langle 44 \rangle - \frac{1}{2} (\langle 21 \rangle \langle 34 \rangle - \langle 23 \rangle \langle 14 \rangle) + \langle 23 \rangle \langle 34 \rangle) \langle 12 \rangle \langle 45 \rangle \dots \langle n1 \rangle$$

$$\frac{6 \pm \sqrt{6^2 - 44}}{21} = \frac{\langle 21 \rangle \langle 34 \rangle - \langle 23 \rangle \langle 14 \rangle \pm \sqrt{(\langle 21 \rangle \langle 34 \rangle - \langle 23 \rangle \langle 14 \rangle)^2 - 4 \langle 21 \rangle \langle 14 \rangle \langle 23 \rangle \langle 34 \rangle}}{2 \langle 21 \rangle \langle 14 \rangle}$$

$$= \frac{1}{2} \left(\frac{\langle 34 \rangle}{\langle 44 \rangle} - \frac{\langle 23 \rangle}{\langle 21 \rangle} \right) \pm \frac{1}{2} (- \dots)$$

$$\begin{aligned} & \langle 21 \rangle \langle 34 \rangle^2 + \langle 23 \rangle^2 \langle 14 \rangle^2 - 2 \langle 23 \rangle \langle 14 \rangle \langle 21 \rangle \langle 34 \rangle - 4 \langle 21 \rangle \langle 14 \rangle \langle 23 \rangle \langle 34 \rangle \\ & = \langle 21 \rangle \langle 34 \rangle^2 + \langle 23 \rangle^2 \langle 14 \rangle^2 - 6 \langle 21 \rangle \langle 23 \rangle \langle 34 \rangle \langle 14 \rangle \end{aligned}$$

no nice factorization

$$\begin{aligned} \hat{A} \hat{B} \hat{C} \hat{D} &= \hat{A} \hat{B} \hat{C} \hat{D} \\ \hat{A} \hat{B} \hat{C} \hat{D} &= \hat{A} \hat{B} \hat{C} \hat{D} \end{aligned}$$

$$(\hat{A} \hat{B} \hat{C} \hat{D}) \hat{E} \hat{F} \hat{G} \hat{H} =$$

$$\hat{A} \hat{B} \hat{C} \hat{D} \hat{E} \hat{F} \hat{G} \hat{H} =$$

$$\hat{A} \hat{B} \hat{C} \hat{D} \hat{E} \hat{F} \hat{G} \hat{H} = \hat{A} \hat{B} \hat{C} \hat{D} \hat{E} \hat{F} \hat{G} \hat{H}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I = \text{identity matrix}$$

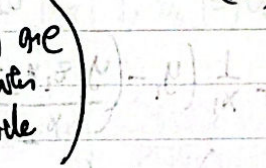
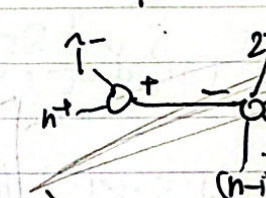
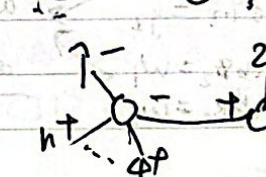
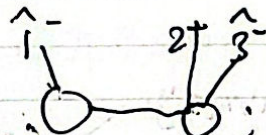
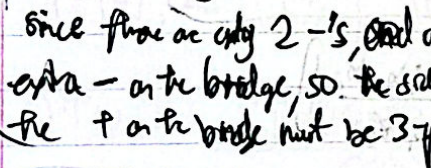
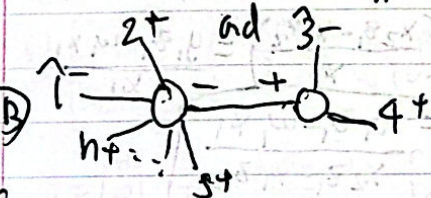
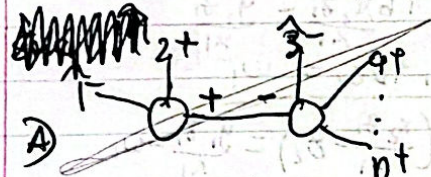
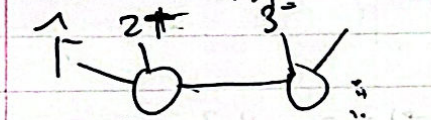
long not notations and last part of A = ...

p57 Why Why (see p174)

WTS: $\hat{p}_1 \hat{p}_2$ - Teller for non-adjacent:

Why $[- \rightarrow \text{on } (1,3)]$

$\hat{A}_n [1^- 2^+ 3^- 4^+ \dots n^+]$



Since there are only 2 -'s, and only one extra - on the bridge, so the side with the + on the bridge must be 3-partite

$$A[\hat{1} \hat{2} \hat{p}_{12}] \frac{1}{p_{12}^2} A[\hat{3} \dots \hat{p}_{12}] + A[\hat{1} \hat{2} \hat{5} \hat{6} \dots \hat{p}_{12}] \frac{1}{p_{12}^2} A[\hat{p}_{12} \hat{3} \hat{4}] \quad B$$

$$C \quad A[\hat{1} \hat{p}_{23} \hat{4} \dots \hat{n}] \frac{1}{p_{23}^2} A[\hat{p}_{23} \hat{2} \hat{3}] + A[\hat{1} \hat{p}_{1n} \hat{n}] \frac{1}{p_{1n}^2} A[\hat{2} \hat{3} \dots \hat{n-1} \hat{p}_{1n}] \quad D$$

$$0 = p_{12}^2 = \langle \hat{1} \hat{2} \rangle \langle \hat{1} \hat{2} \rangle = \langle \hat{1} \hat{2} \rangle \langle \hat{1} \hat{2} \rangle$$

0 (top d 56)

there 2 varn

$$= \frac{\langle \hat{1} \hat{p}_{34} \rangle^4}{\langle \hat{1} \hat{2} \rangle \langle \hat{2} \hat{p}_{34} \rangle \langle \hat{p}_{34} \hat{5} \rangle \dots \langle \hat{n} \hat{1} \rangle} \frac{1}{\langle \hat{3} \hat{4} \rangle \langle \hat{3} \hat{4} \rangle} \frac{[\hat{4} \hat{p}_{34}]^3}{[\hat{4} \hat{3}] [\hat{p}_{34} \hat{3}]} \quad B$$

$$+ \frac{\langle \hat{1} \hat{p}_{23} \rangle^4}{\langle \hat{1} \hat{p}_{23} \rangle \langle \hat{p}_{23} \hat{4} \rangle \langle \hat{4} \hat{5} \rangle \dots \langle \hat{n} \hat{1} \rangle} \frac{1}{\langle \hat{2} \hat{3} \rangle \langle \hat{2} \hat{3} \rangle} \frac{[\hat{2} \hat{p}_{23}]^3}{[\hat{2} \hat{3}] [\hat{p}_{23} \hat{3}]} \quad C$$

then term is almost the same as (3.21) except that $\hat{p}_{12}$ is on the 3

$$\langle \hat{1} \hat{p}_{12} \rangle [\hat{2} \hat{p}_{23}] = -\langle \hat{1} \hat{p}_{12} \rangle [\hat{p}_{12} \hat{2}] = \langle \hat{1} | \hat{p}_{12} | \hat{2} \rangle = \langle \hat{1} | \hat{p}_2 + \hat{p}_3 | \hat{2} \rangle = \langle \hat{1} | \hat{p}_3 | \hat{2} \rangle$$

$$= -\langle \hat{1} \hat{3} \rangle [\hat{3} \hat{2}] = -\langle \hat{1} \hat{3} \rangle [\hat{3} \hat{2}]$$

$$\langle \hat{p}_{23} \hat{4} \rangle [\hat{p}_{23} \hat{3}] = -\langle \hat{4} \hat{2} \rangle [\hat{4} \hat{3}] = \langle \hat{4} | \hat{p}_2 + \hat{p}_3 | \hat{3} \rangle = \langle \hat{4} | \hat{p}_2 | \hat{3} \rangle = \langle \hat{4} \hat{2} \rangle [\hat{2} \hat{3}]$$

$$\Rightarrow \frac{\langle \hat{1} | \hat{P}_{23} \rangle^4}{\langle \hat{1} | \hat{P}_{23} \rangle \langle \hat{P}_{23} | \hat{4} \rangle \dots \langle \hat{n1} \rangle} \frac{1}{\langle 23 \rangle \langle 23 \rangle} \frac{[\langle 2 | \hat{P}_{23} \rangle]^3}{[\langle 23 \rangle] [\langle \hat{P}_{23} | \hat{3} \rangle]}$$

$$= \frac{\langle \hat{1} | \hat{P}_{23} \rangle^3 [\langle 2 | \hat{P}_{23} \rangle]^3}{\langle \hat{P}_{23} | \hat{4} \rangle \langle \hat{P}_{23} | \hat{3} \rangle \langle 45 \rangle \langle 56 \rangle \dots \langle \hat{n1} \rangle \langle 23 \rangle \langle 23 \rangle \langle 23 \rangle}$$

$$= \frac{\langle 13 \rangle^3 [\langle 32 \rangle]^3}{\langle 24 \rangle \langle 32 \rangle \langle 45 \rangle \dots \langle \hat{n1} \rangle \langle 23 \rangle \langle 23 \rangle \langle 23 \rangle}$$

$$= \frac{\langle 13 \rangle^3}{\langle 23 \rangle \langle 24 \rangle \langle 45 \rangle \dots \langle \hat{n1} \rangle}$$

B) $\langle \hat{1} | \hat{P}_{34} \rangle [\hat{P}_{34} | \hat{4}] = \langle \hat{1} | \hat{P}_3 + \hat{P}_4 | \hat{4} \rangle = \langle \hat{1} | \hat{P}_3 | \hat{4} \rangle = \langle \hat{1} | \hat{3} \rangle [\hat{3} | \hat{4}] = \langle 13 \rangle \langle 34 \rangle$

$\langle 2 | \hat{P}_{34} \rangle [\hat{P}_{34} | \hat{3}] = \langle 2 | \hat{P}_3 + \hat{P}_4 | \hat{3} \rangle = \langle 2 | \hat{P}_4 | \hat{3} \rangle = \langle 24 \rangle [\hat{4} | \hat{3}] = \langle 24 \rangle \langle 43 \rangle$

or $\langle 5 | \hat{P}_{34} \rangle [\hat{P}_{34} | \hat{3}] = \dots = \langle 54 \rangle \langle 43 \rangle ?$

we get: $\frac{\langle \hat{1} | \hat{P}_{34} \rangle}{\langle 5 | \hat{P}_{34} \rangle} \cdot \frac{\langle 13 \rangle^3 [\langle 34 \rangle]^3}{\langle 12 \rangle \langle 24 \rangle \langle 43 \rangle \langle 56 \rangle \dots \langle \hat{n1} \rangle \langle 34 \rangle \langle 34 \rangle \langle 34 \rangle} \cdot \frac{\langle 14 \rangle \langle 45 \rangle}{\langle 95 \rangle}$ not equal?

$$= \frac{\langle \hat{1} | \hat{P}_{34} \rangle}{\langle 5 | \hat{P}_{34} \rangle} \cdot \frac{\langle 13 \rangle^3}{\langle 12 \rangle \langle 24 \rangle \langle 34 \rangle \langle 56 \rangle \dots \langle \hat{n1} \rangle}$$

WANT: $\frac{\langle 13 \rangle^3}{\langle 23 \rangle \langle 24 \rangle \langle 45 \rangle \dots \langle \hat{n1} \rangle} + \frac{A}{B} \frac{\langle 13 \rangle^3}{\langle 12 \rangle \langle 24 \rangle \langle 34 \rangle \langle 56 \rangle \dots \langle \hat{n1} \rangle} = \frac{\langle 13 \rangle^3}{\langle 12 \rangle \langle 23 \rangle \dots \langle \hat{n1} \rangle}$

$$\frac{\langle 12 \rangle \langle 34 \rangle}{\langle 24 \rangle} + \frac{A}{B} \frac{\langle 23 \rangle \langle 45 \rangle}{\langle 24 \rangle} = 1$$

read a part of (12)

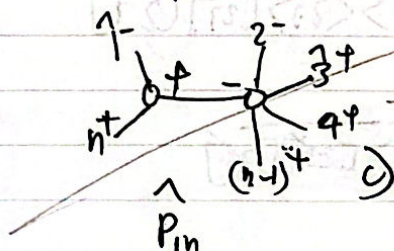
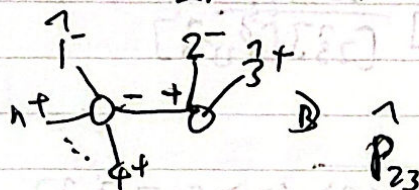
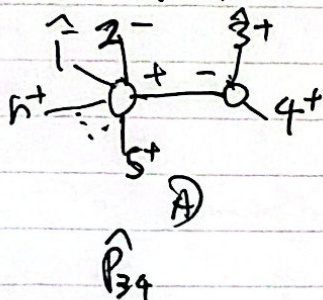
$$\langle 12 \rangle \langle 34 \rangle = \langle 13 \rangle \langle 24 \rangle + \langle 14 \rangle \langle 32 \rangle$$

$$\langle 12 \rangle \langle 34 \rangle + X \langle 23 \rangle \langle 45 \rangle = \langle 24 \rangle \rightarrow X = \frac{\langle 24 \rangle - \langle 12 \rangle \langle 34 \rangle}{\langle 23 \rangle \langle 45 \rangle}$$

$$\frac{\langle \hat{1} | \hat{P}_{34} \rangle}{\langle 5 | \hat{P}_{34} \rangle} \cdot \frac{[\hat{P}_{34} | \hat{3}]}{[\hat{P}_{34} | \hat{3}]} = \frac{\langle 14 \rangle \langle 43 \rangle}{\langle 54 \rangle \langle 43 \rangle}$$

$$\frac{\langle \hat{1} | \hat{P}_{34} \rangle}{\langle 5 | \hat{P}_{34} \rangle} = A/B$$

? Is $[-+] \text{ or } [3]$ correct? $A_n [1^- 2^- 3^+ 4^+ \dots n^+]$



This diagram vanishes by the argument on p56

$$A) \quad A[1^- 2^- \hat{P}_{34}^+ 5^+ 6^+ \dots n^+] \frac{1}{P_{34}^2} A[\hat{P}_{34}^- 3^+ 4^+] + A[1^- \hat{P}_{23}^+ 4^+ 5^+ \dots n^+] \frac{1}{P_{23}^2} A_n[2^- 3^+ \hat{P}_{23}^-]$$

$$\frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 2 \hat{P}_{34} \rangle \langle \hat{P}_{34} 5 \rangle \dots \langle n1 \rangle} \frac{1}{\langle 34 \rangle \langle 34 \rangle} \frac{[34]^3}{[3 \hat{P}_{34}] [4 \hat{P}_{34}]} + \frac{\langle 1 \hat{P}_{23} \rangle^4}{\langle 1 \hat{P}_{23} \rangle \langle \hat{P}_{23} 4 \rangle \dots \langle n1 \rangle} \frac{1}{\langle 23 \rangle \langle 23 \rangle} \frac{[\hat{P}_{23} 3]^3}{[23] [2 \hat{P}_{23}]}$$

$$A) \quad \begin{aligned} \langle 2 \hat{P}_{34} \rangle [\hat{P}_{34} 3] &= \langle 24 \rangle [43] \\ \langle 5 \hat{P}_{34} \rangle [\hat{P}_{34} 4] &= \langle 53 \rangle [34] \end{aligned} \quad \text{or calculate do:} \quad \begin{aligned} \langle 23 \rangle [34] \\ \langle 54 \rangle [43] \end{aligned}$$

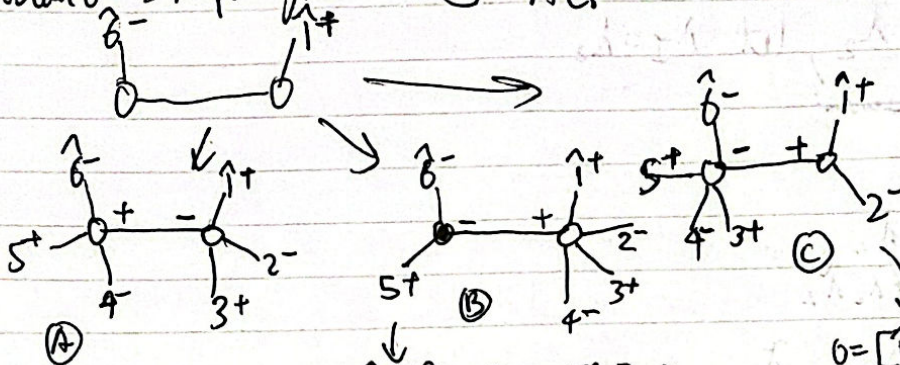
$$\frac{\langle 12 \rangle^4 [34]^3}{\langle 12 \rangle \langle 23 \rangle [34] [34] \langle 45 \rangle \langle 56 \rangle \dots \langle n1 \rangle \langle 34 \rangle [34]} = \frac{\langle 12 \rangle^3}{\langle 12 \rangle \langle 23 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

$$B) \quad \langle 1 \hat{P}_{23} \rangle [\hat{P}_{23} 3] = \langle 12 \rangle [23], \quad \langle \hat{P}_{23} 4 \rangle [\hat{P}_{23} 2] = \langle 43 \rangle [32]$$

$$\frac{\langle 12 \rangle^3 [23]^3}{\langle 43 \rangle [32] \langle 45 \rangle \dots \langle n1 \rangle \langle 23 \rangle [23] [23]} = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

so I actually got Parke-Taylor in both: 2

Problem 6: Six-point gluon scattering $A[1^+ 2^- 3^+ 4^- 5^+ 6^-]$



$$[6, 1] = [-+] \\ \hat{6} = \hat{6} + z\hat{1} \\ \hat{1} = \hat{1} - z\hat{6}$$

(56 BW) $[65]\langle 65 \rangle = 0 \Rightarrow [65] = 0$
 $[65]\langle 65 \rangle$ so must have $\frac{1}{4}$
 from $\langle 56 \rangle$ dependence

$$0 = [12]\langle 12 \rangle = [12]\langle 12 \rangle \Rightarrow \langle 12 \rangle = 0 \\ \Rightarrow \text{must depend on } [12]$$

Matched (C) on p 111-112

$$\frac{[35]^4}{[6\hat{p}_2][\hat{p}_2 3][34][45][56]} \cdot \frac{1}{[12]\langle 12 \rangle} \cdot \frac{[1\hat{p}_2]^3}{[12][\hat{p}_2 2]}$$

$$0 = \hat{p}_{12}^2 = \langle 12 \rangle [12] = \langle 12 \rangle [12] = (\langle 1 - z(6) 2 \rangle) [12] = (\langle 12 \rangle - z\langle 62 \rangle) [12] = 0 \\ z = \frac{\langle 12 \rangle}{\langle 62 \rangle}$$

$$\hat{p}_{12} = [\hat{p}_1 + p_2] = \hat{p}_1 + p_2 = \hat{1} + 2 = [1+2]$$

$$\hat{p}_{12} = \hat{p}_1 + p_2 = \hat{1} + 2 = (1 - z(6)) [1+2] = [1] - z[6] + 2[2]$$

$$= p_1 + p_2 - z(6)[1] = p_1 + p_2 - \frac{42}{26} [1] \text{ ? } \text{Answer: No get } \hat{1} = \frac{(p_1 + p_2)[6]}{\langle 26 \rangle} \text{ ? } \text{ p 110 top of next page}$$

Use EH rule: $\frac{[1\hat{p}_{12}]^3}{[6\hat{p}_2][\hat{p}_2 3][\hat{p}_{12}]} \cdot \frac{\langle \hat{p}_{12} 6 \rangle^3}{\langle \hat{p}_{12} 6 \rangle^3} = \frac{[12]^3 \langle 26 \rangle^3 [35]^4}{[12]\langle 16 \rangle [13]\langle 16 \rangle + [23]\langle 26 \rangle ([16]\langle 16 \rangle + [26]\langle 26 \rangle)}$

$$[1\hat{p}_{12}]\langle 16 \rangle = [12]\langle 26 \rangle$$

$$[3\hat{p}_{12}]\langle 16 \rangle = [31]\langle 16 \rangle + [32]\langle 26 \rangle$$

$$[2\hat{p}_{12}]\langle 16 \rangle = [21]\langle 16 \rangle$$

$$[6\hat{p}_{12}]\langle 16 \rangle = -2\hat{p}_6 \cdot \hat{p}_{12} = +2\hat{p}_6 \cdot (p_3 + p_4 + p_5 + \hat{p}_6) = 2\hat{p}_6(p_3 + p_4 + p_5) = 2(\hat{p}_6 p_3 + \hat{p}_6 p_4 + \hat{p}_6 p_5)$$

$$= (p_3 + p_4 + p_5 + \hat{p}_6)^2 - (p_3 + p_4 + p_5)^2 = \hat{p}_{12}^2 - (p_3 + p_4 + p_5)^2 = -(p_3 + p_4 + p_5)^2$$

$$2(p_6 p_3 + p_6 p_4 + p_6 p_5 + p_3 p_4 + p_3 p_5 + p_4 p_5)$$

$$= [61]\langle 16 \rangle + [62]\langle 26 \rangle$$

$$= [61]\langle 16 \rangle + [62]\langle 26 \rangle$$

$$\hat{p}_{16}^2$$

$$\hat{p}_{26}^2$$

FAIL

Now let, for $\lambda_I \propto \lambda_P \propto \lambda_2$, let $\lambda_I = \lambda_2$

Solve for $\tilde{\lambda}_I$ by method Casimir:

$$\lambda_I \tilde{\lambda}_I = \lambda_P \tilde{\lambda}_1 + \lambda_2 \tilde{\lambda}_2$$

$$\lambda_2 \tilde{\lambda}_I = (\lambda_1 + \epsilon \lambda_6) \tilde{\lambda}_1 + \lambda_2 \tilde{\lambda}_2$$

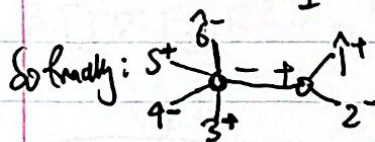
$$= \lambda_1 \tilde{\lambda}_1 + \frac{42}{62} \lambda_6 \tilde{\lambda}_1 + \lambda_2 \tilde{\lambda}_2$$

$$= \frac{(\langle 62 \rangle \lambda_1 + \langle 12 \rangle \lambda_6)}{\langle 62 \rangle} \tilde{\lambda}_1 + \lambda_2 \tilde{\lambda}_2$$

Schrodinger $\Rightarrow$

$$= \frac{\langle 61 \rangle \lambda_2 \tilde{\lambda}_1}{\langle 62 \rangle} + \lambda_2 \tilde{\lambda}_2$$

$$\Rightarrow \tilde{\lambda}_I = \frac{\langle 61 \rangle \tilde{\lambda}_1}{\langle 62 \rangle} + \tilde{\lambda}_2 = \frac{\langle 61 \rangle \tilde{\lambda}_1 + \langle 62 \rangle \tilde{\lambda}_2}{\langle 62 \rangle} = \frac{(p_1 + p_2) | 6 \rangle}{\langle 62 \rangle}$$



$$= \frac{[35]^2}{[13][24][35][46][57]} \cdot \left\{ \frac{1}{[12]\langle 12 \rangle} \cdot \frac{[11]^3}{[12][12]} \right\}$$

Now: take $\tilde{\lambda}_I - \frac{\langle 61 \rangle}{\langle 62 \rangle} \tilde{\lambda}_1 - \tilde{\lambda}_2 = 0$ constant with:

$$[1]: [11] = [2]$$

$$[2]: [12] = \frac{\langle 61 \rangle}{\langle 62 \rangle} [12]$$

$$\frac{[12]^3}{[12]^2 \langle 12 \rangle} \cdot \frac{1}{\frac{\langle 61 \rangle}{\langle 62 \rangle} [12]} = \frac{\langle 62 \rangle}{\langle 61 \rangle \langle 12 \rangle} \rightarrow \frac{1}{[12]\langle 12 \rangle} \cdot \frac{[12]^3}{\frac{\langle 61 \rangle}{\langle 62 \rangle} [12][12]} = -\frac{\langle 62 \rangle}{\langle 12 \rangle \langle 61 \rangle}$$

$$= \frac{[35]^2}{[13][24][35][46][57]} \cdot \frac{\langle 62 \rangle}{\langle 61 \rangle \langle 12 \rangle}$$

$$\frac{[5 | p_6 + p_1 | 12]}{\langle 62 \rangle} \cdot \frac{\langle 2 | (p_1 + p_6) (p_1 + p_2) | 6 \rangle}{\langle 62 \rangle} \cdot \frac{\langle 6 | p_1 + p_2 | 3 \rangle}{\langle 26 \rangle}$$

$$\frac{1}{\langle 62 \rangle} ([56]\langle 62 \rangle + [51]\langle 12 \rangle) \cdot \langle 2 | p_1^2 + p_1 p_2 + p_6 p_1 + p_6 p_2 | 6 \rangle \cdot (\langle 61 \rangle [13] + \langle 62 \rangle [23])$$

$$\langle 21 \rangle [12] \langle 26 \rangle + \langle 26 \rangle [61] \langle 16 \rangle + \langle 26 \rangle [62] \langle 26 \rangle$$

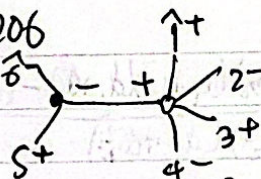
$$= \langle 26 \rangle (p_1 p_2 + p_1 p_6 + p_2 p_6) (-1)$$

$$= \langle 62 \rangle (p_1 + p_2 + p_6)^2$$

$$\rightarrow \frac{-2 [35]^2 \langle 62 \rangle}{\langle 61 \rangle \langle 12 \rangle [13][24][35]} \cdot \frac{\langle 62 \rangle}{\langle 61 \rangle \langle 12 \rangle} \cdot \frac{1}{(p_1 + p_2 + p_6)^2} [5 | p_6 + p_1 | 12] \langle 6 | p_1 + p_2 | 3 \rangle$$

BCFW out from p206

Next diagram: (B)



$$\frac{\langle 61 \rangle^3}{\langle 65 \rangle \langle 15 \rangle} \cdot \frac{1}{[56][56]} \cdot \frac{\langle 24 \rangle^2}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle \langle 11 \rangle}$$

$$p_6^2 = [56][56] = 0 \rightarrow \begin{aligned} \langle 56 \rangle [5(6+1)] &= 0 \\ \langle 56 \rangle ([56] + 2[51]) &= 0 \\ z &= -\frac{[56]}{[51]} \end{aligned}$$

$$\langle 1 \rangle = \langle 1 - z \langle 6 \rangle = \langle 1 + \frac{[56]}{[51]} \langle 6 \rangle = \frac{[51] \langle 1 \rangle + [56] \langle 6 \rangle}{[51]} = \frac{[51] (p_1 + p_6)}{[51]}$$

We know $p_1 = p_5 + p_6$

$$\lambda_1 \tilde{\lambda}_1 = \lambda_5 \tilde{\lambda}_5 + \lambda_6 \tilde{\lambda}_6 \xrightarrow[\text{solve for } \lambda_1]{\text{square } \tilde{\lambda}_1 = \tilde{\lambda}_5} \lambda_1 \tilde{\lambda}_5 = \lambda_5 \tilde{\lambda}_5 + \lambda_6 (\tilde{\lambda}_6 + \frac{[56]}{[51]} \tilde{\lambda}_1)$$

$$= \lambda_5 \tilde{\lambda}_5 + \lambda_6 \tilde{\lambda}_6 - \frac{[56]}{[51]} \lambda_6 \tilde{\lambda}_1$$

$$\text{solve: } [56] \tilde{\lambda}_1 - [16] \tilde{\lambda}_5 + [15] \tilde{\lambda}_6 = 0$$

$$[51] \tilde{\lambda}_6 - [56] \tilde{\lambda}_1 = -[16] \tilde{\lambda}_5$$

$$= [61] \tilde{\lambda}_5$$

$$\frac{[51] \lambda_6 \tilde{\lambda}_6 - [56] \lambda_6 \tilde{\lambda}_1}{[51]} = \frac{\lambda_6 ([51] \tilde{\lambda}_6 - [56] \tilde{\lambda}_1)}{[51]} = \frac{[61]}{[51]} \tilde{\lambda}_5 \lambda_6$$

$$\Rightarrow \lambda_1 = \lambda_5 + \frac{[61]}{[51]} \lambda_6$$

$$= \frac{[51] \lambda_5 + [61] \lambda_6}{[51]} = \frac{(p_5 + p_6) \cdot 1}{[51]}$$

$$\langle \lambda_1 6 \rangle = \langle 56 \rangle$$

$$\langle \lambda_1 5 \rangle = \frac{[61]}{[51]} \langle 65 \rangle$$

$$\text{so } \frac{\langle 61 \rangle^3}{\langle 65 \rangle \langle 15 \rangle} \cdot \frac{1}{[56][56]} = \frac{\langle 56 \rangle^3}{\langle 65 \rangle \frac{[61]}{[51]} \langle 65 \rangle [56][56]} = \frac{[51]}{[61][56]}$$

$$\langle 12 \rangle \langle 41 \rangle \langle 11 \rangle$$

$$= \frac{[51] (p_1 + p_6) \cdot 1}{[51]} \cdot \frac{\langle 41 \rangle (p_5 + p_6) \cdot 1}{[51]} \cdot \frac{[1] (p_5 + p_6) \cdot 1}{[51]^2}$$

$$[1] p_5 p_1 + p_5 p_6 + p_6 p_1 + p_6^2 [5] = [5] \langle 51 \rangle [5] + [5] \langle 56 \rangle [6] + [6] \langle 61 \rangle [5]$$

$$= [5] (p_1 \cdot p_5 + p_5 \cdot p_6 - p_1 \cdot p_6)$$

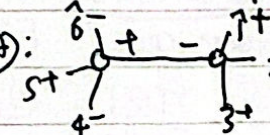
$$= \frac{[5]}{2} (p_1 + p_5 + p_6)^2$$

$$\frac{[51] (p_1 + p_6) \cdot 1}{[51]^4} \cdot \frac{\langle 41 \rangle (p_5 + p_6) \cdot 1}{[51]} \cdot \frac{[51]}{2} (p_1 + p_5 + p_6)^2$$

$$\textcircled{1} \frac{[35]^2 [26]^2}{\langle 61 \rangle \langle 42 \rangle [34] [45] (p_6 + p_1 + p_3) \langle 61 p_1 p_3 | [51 p_6 + p_1] \rangle}$$

↑+2

$$\rightarrow = \frac{[51]^2}{[61][56] \langle 23 \rangle \langle 34 \rangle [51 p_1 + p_6 | 2] \langle 41 p_3 + p_6 | 1 \rangle (p_1 + p_3 + p_6)^2} [51]^2 = \frac{[24]^2 [51]^2}{[61][56] \langle 23 \rangle \langle 34 \rangle [51 p_1 + p_6 | 2] \langle 41 p_3 + p_6 | 1 \rangle (p_1 + p_3 + p_6)^2}$$

Diagram (A):  = left: $\frac{\langle 61 \rangle^2}{\langle 61 \rangle \langle 14 \rangle \langle 45 \rangle \langle 56 \rangle}$ or $\frac{[51]^2}{[65][14][45][56]}$

right: $\frac{\langle 12 \rangle^2}{\langle 11 \rangle \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle}$ or $\frac{[13]^2}{[11][12][23][31]}$

$$\frac{1}{p_1^2} = \frac{1}{(p_1^2 + p_2^2 + p_3^2)} = \frac{1}{(p_1^2 + p_2^2 + p_6^2)} = \frac{2}{p_1 p_2 + p_1 p_3 + p_2 p_3}$$

Remark: don't have to deal with shift

$$\frac{\langle 61 \rangle^2}{\langle 61 \rangle \langle 14 \rangle \langle 45 \rangle \langle 56 \rangle} \cdot \frac{[13]^2}{[11][12][23][31]} \cdot \frac{1}{(p_1^2 + p_2^2 + p_3^2)^2}$$

still need $\hat{1}, \hat{1}$

what's the λ comp of $\hat{p}_1 + \hat{p}_2 + \hat{p}_3 = \hat{p}_5 = \lambda \hat{1} \hat{1} = p_1 + p_2 + p_6$

$$(1 + z \langle 6 \rangle) [1] + z \langle 2 \rangle [3] = 4 \langle 4 \rangle [4 + 5] [5] + 6 \langle 6 - z \rangle [1]$$

$$1 [1] + z \langle 2 \rangle [3] [3] + z \langle 6 \rangle [1] = 4 \langle 4 \rangle [4 + 5] [5] + 6 \langle 6 - z \rangle [1]$$

$$\begin{aligned} & \langle 6 \rangle [1] \\ & \langle 62 \rangle [2] [1] + \langle 63 \rangle [3] [1] = \langle 64 \rangle [4] [1] + \langle 65 \rangle [5] [1] \end{aligned}$$

$$\langle 12 \rangle [26] + \langle 13 \rangle [36] + z \langle 16 \rangle [16] = \langle 14 \rangle [46] + \langle 15 \rangle [56] - z \langle 16 \rangle [16]$$

$$\Rightarrow z = \frac{\langle 14 \rangle [46] + \langle 15 \rangle [56] - \langle 12 \rangle [26] - \langle 13 \rangle [36]}{2 p_1 \cdot p_6}$$

$$\hat{p}_5^2 = 0: (\hat{p}_1 + \hat{p}_2 + \hat{p}_3)(\hat{p}_1 + \hat{p}_2 + \hat{p}_3) = 0$$

$$= 2(\hat{p}_1 \cdot \hat{p}_2 + \hat{p}_1 \cdot \hat{p}_3 + \hat{p}_2 \cdot \hat{p}_3) = 0$$

$$[12] \langle 12 \rangle + [13] \langle 13 \rangle + [23] \langle 23 \rangle = 0, \text{ also } [45] \langle 45 \rangle + [46] \langle 46 \rangle + [56] \langle 56 \rangle = 0$$

$$[12] (1 - z \langle 6 \rangle) [2] + [13] (1 - z \langle 6 \rangle) [3] + [23] \langle 23 \rangle = 0$$

$$[12] (12 - z \langle 62 \rangle) + [13] (13 - z \langle 63 \rangle) + [23] \langle 23 \rangle = 0$$

$$[12] \langle 12 \rangle - z [12] \langle 62 \rangle + [13] \langle 13 \rangle - z [13] \langle 63 \rangle + [23] \langle 23 \rangle = 0$$

$$z = \frac{p_1 \cdot p_2 + p_1 \cdot p_3 + p_2 \cdot p_3}{2([12] \langle 62 \rangle - [13] \langle 63 \rangle)}$$

$$2([12] \langle 62 \rangle - [13] \langle 63 \rangle)$$

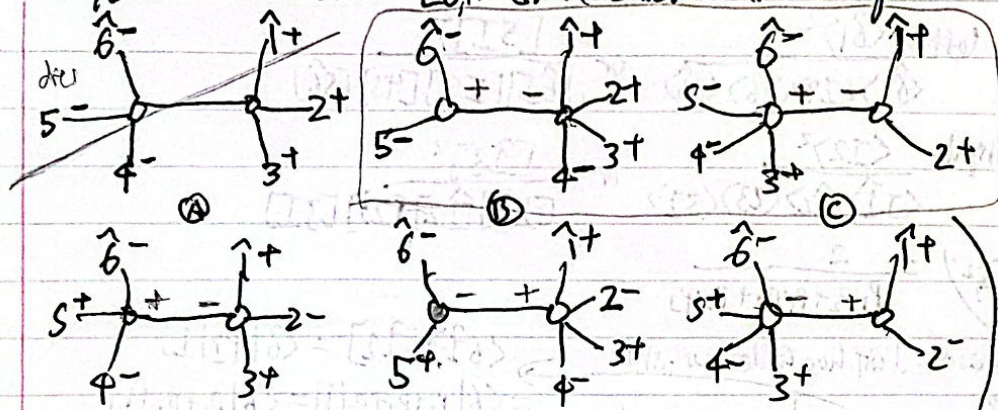
$$= \frac{\langle 61 \rangle^2 [13]^2}{[12][23] \langle 45 \rangle \langle 56 \rangle \langle 41 p_2 + p_3 | 1 \rangle \langle 61 p_1 + p_3 | 3 \rangle (p_1 + p_2 + p_3)^2}$$

$$\begin{aligned} & [45] \langle 45 \rangle + [46] \langle 46 \rangle + [56] \langle 56 \rangle = 0 \\ & [45] \langle 45 \rangle + [4(6+z1)] \langle 46 \rangle + [5(6+z1)] \langle 56 \rangle = 0 \\ & [45] \langle 45 \rangle + ([46] + z[41]) \langle 46 \rangle + ([56] + z[51]) \langle 56 \rangle = 0 \\ & [45] \langle 45 \rangle + [46] \langle 46 \rangle + z[41] \langle 46 \rangle + [56] \langle 56 \rangle + z[51] \langle 56 \rangle = 0 \end{aligned}$$

$$z = - \frac{p_1 \cdot p_2 + p_1 \cdot p_3 + p_2 \cdot p_3}{2([41] \langle 46 \rangle + [51] \langle 56 \rangle)}$$

Do $A[1+2+3+4-5-6]$ follow Energy-Hungry

First, what if we did the $[6]$ (like as before)? Diagrams



only 2 diagrams for this one, but almost the same as before

$$\textcircled{B} \frac{\langle 65 \rangle^3}{\langle 61 \rangle \langle 51 \rangle} \frac{1}{\langle 56 \rangle \langle 66 \rangle} \frac{\langle 14 \rangle^2}{\langle 11 \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$$

$$\begin{aligned} \hat{p}_{56}^2 &= \langle 56 \rangle \langle 66 \rangle = \langle 56 \rangle \langle 66 \rangle = 0 \\ 0 &= \langle 56 \rangle = \langle 5(6+21) \rangle = \langle 56 \rangle + 2 \langle 51 \rangle = 0 \\ z &= -\frac{\langle 56 \rangle}{\langle 51 \rangle} \end{aligned}$$

$$\begin{aligned} \langle 1 &= \langle 1 - z \langle 6 \rangle = \langle 1 + \frac{\langle 66 \rangle}{\langle 51 \rangle} \langle 6 \rangle \\ &= \frac{\langle 51 \rangle \langle 1 + \frac{\langle 66 \rangle}{\langle 51 \rangle} \langle 6 \rangle}{\langle 51 \rangle} = \frac{\langle 51 \rangle \hat{p}_1 + \hat{p}_6}{\langle 51 \rangle} \end{aligned}$$

$$\text{From before: } \hat{1} = \frac{\langle \hat{p}_5 + \hat{p}_6 \rangle \hat{1}}{\langle 51 \rangle} = \hat{5} + \frac{\langle 61 \rangle}{\langle 51 \rangle} \hat{6}$$

$$\begin{aligned} \langle 61 \rangle &= \langle 65 \rangle \\ \langle 51 \rangle &= \frac{\langle 61 \rangle}{\langle 51 \rangle} \langle 56 \rangle \end{aligned}$$

$$\frac{\langle 65 \rangle^3}{\langle 61 \rangle \langle 51 \rangle} \frac{1}{\langle 56 \rangle \langle 66 \rangle} = \frac{\langle 65 \rangle^3 \langle 51 \rangle}{\langle 65 \rangle \langle 56 \rangle \langle 61 \rangle \langle 56 \rangle \langle 66 \rangle} = \frac{\langle 51 \rangle}{\langle 61 \rangle \langle 56 \rangle} \leftarrow \text{same as before}$$

$$\text{Now: } \frac{\langle 14 \rangle^2}{\langle 11 \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle} = \left(\langle 45 \rangle + \frac{\langle 61 \rangle}{\langle 51 \rangle} \langle 46 \rangle \right)^2$$

$$\begin{aligned} 0 | \hat{p}_5 \hat{p}_1 + \hat{p}_5 \hat{p}_6 + \hat{p}_6 \hat{p}_1 + \hat{p}_6 \hat{p}_6 | \hat{5} &= \langle 51 \rangle \langle 51 \rangle \langle 45 \rangle + \langle 51 \rangle \langle 56 \rangle \langle 65 \rangle + \langle 61 \rangle \langle 61 \rangle \langle 45 \rangle \\ &= \langle 51 \rangle (\hat{p}_1 \hat{p}_5 - \hat{p}_5 \hat{p}_6 - \hat{p}_1 \hat{p}_6) = -\frac{\langle 51 \rangle}{2} (\hat{p}_1 + \hat{p}_5 + \hat{p}_6)^2 \end{aligned}$$

$$\textcircled{C} \frac{\langle 12 \rangle^3}{\langle 11 \rangle \langle 21 \rangle} \frac{1}{\langle 42 \rangle \langle 12 \rangle} \frac{\langle 13 \rangle^2}{\langle 13 \rangle \langle 31 \rangle \langle 43 \rangle \langle 61 \rangle \langle 61 \rangle}$$

$$\begin{aligned} \hat{p}_{12}^2 &= \langle 12 \rangle \langle 22 \rangle = \langle 12 \rangle \langle 22 \rangle = 0 \\ 0 &= \langle 12 \rangle = \langle (1 - z \langle 6 \rangle) \langle 2 \rangle = \langle 12 \rangle - z \langle 62 \rangle \\ z &= \frac{\langle 12 \rangle}{\langle 62 \rangle} \end{aligned}$$

$$\begin{aligned} \hat{2} &= \hat{2} + z \hat{1} = \hat{2} + \frac{\langle 12 \rangle}{\langle 62 \rangle} \hat{1} = \frac{\langle 62 \rangle \hat{2} + \langle 12 \rangle \hat{1}}{\langle 62 \rangle} \\ &= \frac{\hat{p}_6 + \hat{p}_1}{\langle 62 \rangle} \end{aligned}$$

$$\hat{1} = \frac{\langle \hat{p}_1 + \hat{p}_2 \rangle \hat{1}}{\langle 62 \rangle} = \frac{\langle 61 \rangle}{\langle 62 \rangle} \hat{1} + \hat{2}$$

$$\begin{aligned} \langle 11 \rangle &= \langle 22 \rangle \\ \langle 21 \rangle &= \frac{\langle 61 \rangle}{\langle 62 \rangle} \langle 21 \rangle \end{aligned}$$

$$\frac{\langle 12 \rangle^3}{\langle 11 \rangle \langle 21 \rangle} \frac{1}{\langle 42 \rangle \langle 12 \rangle} = \frac{\langle 12 \rangle^3 \langle 62 \rangle}{\langle 61 \rangle \langle 61 \rangle \langle 21 \rangle \langle 42 \rangle \langle 12 \rangle} = \frac{\langle 62 \rangle}{\langle 61 \rangle \langle 12 \rangle}$$

So far (b) we get:

$$\begin{aligned}
 & \frac{[51]}{[61][56]} \cdot \left(\langle 45 \rangle + \frac{[61]}{[51]} \langle 46 \rangle \right) \cdot \frac{[51]^3}{[51] (p_1 + p_3 + p_6)^2 [5(p_1 + p_6)2] \langle 4(p_1 + p_6)1 \rangle} \\
 &= \frac{\langle 45 \rangle}{\langle 23 \rangle \langle 34 \rangle} \cdot \frac{[51]^4}{[61][56] (p_1 + p_3 + p_6)^2 [5(p_1 + p_6)2] \langle 4(p_1 + p_6)1 \rangle} + \frac{\langle 46 \rangle}{\langle 23 \rangle \langle 34 \rangle} \cdot \frac{[51]^3}{[56] (p_1 + p_3 + p_6)^2 [5(p_1 + p_6)2] \langle 4(p_1 + p_6)1 \rangle} \\
 &= \frac{[51]^3}{[56] \langle 23 \rangle \langle 34 \rangle} \cdot \frac{1}{(p_1 + p_3 + p_6)^2 [5(p_1 + p_6)2] \langle 4(p_1 + p_6)1 \rangle} \left(\frac{\langle 45 \rangle [51]}{[61]} + \langle 46 \rangle \right) \\
 & \Rightarrow - \frac{[51]^3}{[61][56] \langle 23 \rangle \langle 34 \rangle} \cdot \frac{1}{(p_1 + p_3 + p_6)^2 [5(p_1 + p_6)2]} \cdot \left(\langle 45 \rangle + \frac{[61]}{[51]} \langle 46 \rangle \right)
 \end{aligned}$$

for (c): $[56][61]$

$$\begin{aligned}
 &= \frac{[5(p_1 + p_6)2]}{\langle 62 \rangle} \cdot \frac{\langle 2(p_1 + p_6)(p_1 + p_2)16 \rangle}{\langle 62 \rangle \langle 62 \rangle} \rightarrow \langle 2(p_1^2 + p_1 p_2 + p_6 p_1 + p_6 p_2)6 \rangle \\
 &= \langle 21 \rangle \langle 12 \rangle \langle 26 \rangle + \langle 26 \rangle \langle 61 \rangle \langle 16 \rangle + \langle 26 \rangle \langle 62 \rangle \langle 26 \rangle \\
 &= \langle 26 \rangle (-p_1 p_2 - p_1 p_6 - p_2 p_6) \\
 &= -\langle 26 \rangle (p_1 + p_2 + p_6)^2 \\
 &= \frac{[5(p_1 + p_6)2]}{\langle 62 \rangle^2} (p_1 + p_2 + p_6)^2
 \end{aligned}$$

so total:
$$-\frac{\langle 62 \rangle}{\langle 61 \rangle \langle 12 \rangle} \frac{1}{[34][45]} \frac{\langle 62 \rangle^2}{[5(p_1 + p_6)2] (p_1 + p_2 + p_6)^2} \left(\frac{[61]}{\langle 62 \rangle} [13] + [23] \right)$$

$$= \langle p_1 p_2 p_1 \rangle, \langle p_1 p_2 p_2 \rangle, \langle p_1 p_2 p_6 \rangle, \dots$$

Chapter 4

$$= \frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)}$$

$$\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)}$$

$$\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)}$$

$$\left[(0.85) \left(\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)} \right) \right] \cdot \frac{1}{2} + \frac{1}{2} \left(\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)} \right)$$

$$\frac{1}{2} \left[\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)} \right] + \frac{1}{2} \left[\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)} \right]$$

$$\frac{1}{2} \left[\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)} \right] + \frac{1}{2} \left[\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)} \right]$$

$$\frac{1}{2} \left[\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)} \right] + \frac{1}{2} \left[\frac{(0.85)(0.15) - (0.15)(0.85)}{(0.85)(0.15) - (0.15)(0.85)} \right]$$

$$(0.85)(0.15) - (0.15)(0.85)$$

$$(0.85)(0.15) - (0.15)(0.85)$$

$$(0.85)(0.15) - (0.15)(0.85)$$

Ex 4.1 Blum Blum
 Check that $\mathcal{L}_0$ in (4.1) is invariant under (4.2) up to total derivative

~~$$\mathcal{L}_0 = i \psi_a^\dagger \bar{\sigma}_{ba}^\mu \partial_\mu \psi_b - \partial_\mu \bar{\phi} \partial^\mu \phi$$~~

$$\delta \mathcal{L}_0 = i (\delta_\epsilon \psi_a^\dagger) \bar{\sigma}^{\mu ab} \partial_\mu (\delta_\epsilon \psi_b) - \partial_\mu (\delta_\epsilon \bar{\phi}) \partial^\mu (\delta_\epsilon \phi)$$

= to linear order in ϵ

$$= i (\delta_\epsilon \psi_a^\dagger) \bar{\sigma}^{\mu ab} \partial_\mu \psi_b + i \psi_a^\dagger \bar{\sigma}_{ba}^\mu \partial_\mu (\delta_\epsilon \psi_b) - \partial_\mu (\delta_\epsilon \bar{\phi}) \partial^\mu \phi - \partial_\mu \bar{\phi} \partial^\mu (\delta_\epsilon \phi)$$

ps 4.1 see:

$$(\sigma^\mu)_{\alpha\beta}, (\bar{\sigma}^\mu)^{\dot{\alpha}\dot{\beta}}$$

$$= i (1 \delta_\epsilon \bar{\phi} \epsilon^c \sigma_{ca}^\mu) \bar{\sigma}^{\mu ab} \partial_\mu \psi_b + i \psi_a^\dagger \bar{\sigma}^{\mu ab} \partial_\mu (-i \sigma_{bc}^\nu \epsilon^{+c} \partial_\nu \phi) - \partial_\mu (\epsilon_a^\dagger \psi^\dagger a) \partial^\mu \phi - \partial_\mu \bar{\phi} \partial^\mu (\epsilon^a \psi_a)$$

$$= -\partial_\mu \bar{\phi} \epsilon^c \sigma_{ca}^\mu \bar{\sigma}^{\mu ab} \partial_\mu \psi_b + \underbrace{\psi_a^\dagger \bar{\sigma}^{\mu ab} \sigma_{bc}^\nu \epsilon^{+c} \partial_\mu \partial_\nu \phi}_{\text{II}} - \epsilon_a^\dagger \partial_\mu \psi^\dagger a \partial^\mu \phi - \epsilon^a \partial_\mu \bar{\phi} \partial^\mu \psi_a$$

$$\partial_\mu (\psi_a^\dagger \bar{\sigma}^{\mu ab} \sigma_{bc}^\nu \epsilon^{+c}) - (\partial_\mu \psi_a^\dagger) \bar{\sigma}^{\mu ab} \sigma_{bc}^\nu \epsilon^{+c} \partial_\nu \phi$$

~~$$\sigma_{ca}^\mu \bar{\sigma}^{\mu ab} = (1, \sigma_x, \sigma_y, \sigma_z) \cdot (1, -\sigma_x, -\sigma_y, -\sigma_z)$$~~

~~$$= 1 - \sigma_x^2 - \sigma_y^2 - \sigma_z^2$$~~

~~$$= 2\mathbb{I} - \mathbb{I} - \mathbb{I} - \mathbb{I} = -2\mathbb{I}$$~~

~~$$= -2\delta_{cb}$$~~

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

~~$$= -\partial_\mu \bar{\phi} \epsilon^b \partial_\mu \psi_b + \partial_\mu (\psi_a^\dagger \epsilon^{+a}) \partial^\mu \phi - \partial_\mu \bar{\phi} \partial^\mu (\epsilon^a \psi_a)$$~~

$$\sigma_\nu \bar{\sigma}_\mu - \bar{\sigma}_\nu \sigma_\mu = g^{\mu\nu}$$

Dirac-Lag Bx 9.1

$$\mathcal{L}_0 = i \psi^\dagger_{\dot{a}} \bar{\sigma}^{\mu}_{\dot{a}b} \partial_\mu \psi_b - \partial_\mu \bar{\phi} \partial^\mu \phi$$

$$\delta_\epsilon \mathcal{L}_0 = 0?$$

$$\delta_\epsilon \mathcal{L}_0 = i \delta_\epsilon \psi^\dagger_{\dot{a}} \bar{\sigma}^{\mu}_{\dot{a}b} \partial_\mu \psi_b + i \psi^\dagger_{\dot{a}} \bar{\sigma}^{\mu}_{\dot{a}b} \partial_\mu \delta_\epsilon \psi_b - \partial_\mu \delta_\epsilon \bar{\phi} \partial^\mu \phi - \partial_\mu \bar{\phi} \partial^\mu \delta_\epsilon \phi$$

$$= i \left(i \partial_\mu \bar{\phi} \epsilon^b \sigma_{ba}^\mu \right) \bar{\sigma}^{\mu}_{\dot{a}c} \partial_\mu \psi_c + i \psi^\dagger_{\dot{a}} \bar{\sigma}^{\mu}_{\dot{a}c} \partial_\mu \left(-i \sigma_{ab}^\mu \epsilon^b \partial_\mu \phi \right) - \partial_\mu (\epsilon^\dagger \psi^\dagger) \partial^\mu \phi - \partial_\mu \bar{\phi} \partial^\mu (\epsilon \psi)$$

These two terms cancel out:

$$- \partial_\mu \bar{\phi} \epsilon^b \sigma_{ba}^\mu \bar{\sigma}^{\mu}_{\dot{a}c} \partial_\mu \psi_c - \partial_\mu \bar{\phi} \epsilon \partial^\mu \psi$$

need to watch out for $\mu=\nu$ terms

These 2 terms should be a total derivative $\leftarrow \partial_\mu (\epsilon^\dagger \psi^\dagger \partial^\mu \phi)$

Elmag - Huang

Ex 9.1 $\delta_\epsilon \mathcal{L}_0 = \int d^4x \left(i \delta_\epsilon \psi^\dagger \bar{\sigma}^\mu \partial_\mu (\delta_\epsilon \psi) + \delta_\mu (\delta_\epsilon \Phi) \partial^\mu \psi - \delta_\mu \Phi \partial^\mu (\delta_\epsilon \psi) \right)$

or $i \delta_\epsilon \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + i \psi^\dagger \bar{\sigma}^\mu \partial_\mu (\delta_\epsilon \psi) - \delta_\mu (\delta_\epsilon \Phi) \partial^\mu \psi - \delta_\mu \Phi \partial^\mu (\delta_\epsilon \psi)$

Integrate by parts (should be straightforward)

$i (i \delta_\mu \Phi \epsilon^b \sigma_{ba}^\mu) \bar{\sigma}^\nu \partial_\nu \psi^c + i \psi^\dagger \bar{\sigma}^\mu \partial_\mu (-i \sigma_{ab}^\mu \epsilon^b \partial_\mu \psi)$

$= -(\partial_\mu \Phi) \epsilon^b \sigma_{ba}^\mu \bar{\sigma}^\nu \partial_\nu \psi^c + \psi^\dagger \bar{\sigma}^\mu \sigma_{ca}^\nu \epsilon^b \partial_\mu \partial_\nu \psi$

$- \partial_\mu (\epsilon^+ \psi^+) \partial^\mu \psi - \delta_\mu \Phi \partial^\mu (\epsilon \psi)$
 $- \epsilon^+ \partial_\mu \psi^\dagger \partial^\mu \psi - \epsilon \delta_\mu \Phi \partial^\mu \psi$
 These equations

$\sigma_{ba}^\mu \bar{\sigma}^\nu = (\bar{\sigma} \sigma)_{bc}^{\mu\nu}$

$(\sigma^\mu \bar{\sigma}^\nu + \sigma^\nu \bar{\sigma}^\mu)_a^b = -2 \eta^{\mu\nu} \delta_a^b$

~~It would be nice to have this identity, but we get~~

Integrate by parts: $\partial_\mu \partial_\nu \Phi \epsilon^b \sigma_{ba}^\mu \bar{\sigma}^\nu \psi^c = \left(\psi^\dagger \bar{\sigma}^\mu \sigma_{ca}^\nu \epsilon^b \partial_\mu \partial_\nu \psi \right)^+$

symmetric in μ and ν

Thus $\sigma_{ca}^\mu \bar{\sigma}^\nu = \sigma_{ca}^\mu \bar{\sigma}^\nu$

Danick's idea:

Let $\sigma^\mu \bar{\sigma}^\nu = \frac{1}{2} \sigma^\mu \bar{\sigma}^\nu + \frac{1}{2} \bar{\sigma}^\mu \sigma^\nu$. We know $\frac{\sigma^\mu \bar{\sigma}^\nu + \sigma^\nu \bar{\sigma}^\mu}{2} + \eta^{\mu\nu} = 0$

Now $-(\partial_\mu \Phi) \epsilon^b \sigma_{ba}^\mu \bar{\sigma}^\nu \partial_\nu \psi^c$

$= -\partial_\mu \Phi \epsilon^b \frac{\sigma_{ba}^\mu \bar{\sigma}^\nu}{2} \partial_\nu \psi^c + -\partial_\mu \Phi \epsilon^b \frac{\bar{\sigma}_{ba}^\mu \sigma^\nu}{2} \partial_\nu \psi^c$

$- \partial_\nu \Phi \epsilon^b \frac{\sigma_{ba}^\mu \bar{\sigma}^\nu}{2} \partial_\mu \psi^c$

$+ \partial_\mu \partial_\nu \Phi \epsilon^b \frac{\sigma_{ba}^\mu \bar{\sigma}^\nu}{2} \psi^c$ $\downarrow$ by parts + total derivative

$+ \partial_\nu \partial_\mu \Phi \epsilon^b \frac{\sigma_{ba}^\mu \bar{\sigma}^\nu}{2} \psi^c$ + total derivative

$- \partial_\mu \Phi \epsilon^b \frac{\sigma_{ba}^\mu \bar{\sigma}^\nu}{2} \partial_\nu \psi^c$ $\downarrow$ by parts

$$\begin{aligned}
 &= -\partial_\mu \bar{\psi} \varepsilon^b \left(\frac{\sigma^\mu \bar{\sigma}^\nu + \sigma^\nu \bar{\sigma}^\mu}{2} \right) \partial_\nu \psi^c + \text{TD} - \varepsilon \partial_\mu \bar{\psi} \delta^\mu \psi \\
 &= -\partial_\mu \bar{\psi} \varepsilon^b \left(\frac{\sigma^\mu \bar{\sigma}^\nu + \sigma^\nu \bar{\sigma}^\mu}{2} \right) \partial_\nu \psi^c + \varepsilon \partial_\mu \bar{\psi} \eta^{\mu\nu} \partial_\nu \psi + \text{TD} \\
 &= -\partial_\mu \bar{\psi} \varepsilon \left(\frac{\sigma^\mu \bar{\sigma}^\nu + \sigma^\nu \bar{\sigma}^\mu}{2} + \eta^{\mu\nu} \right) \partial_\nu \psi + \text{TD} \\
 &= 0
 \end{aligned}$$

similar with the other piece, except we have

$$\oplus \psi^\dagger \bar{\sigma}^\mu \sigma^\nu \varepsilon^\dagger \partial_\mu \partial_\nu \phi \ominus \varepsilon^\dagger \partial_\mu \psi^\dagger \partial^\mu \phi$$

different

but this is exactly what we need, since

$$\bar{\sigma}^\mu \sigma^\nu = \frac{1}{2} (\bar{\sigma}^\mu \sigma^\nu + \bar{\sigma}^\nu \sigma^\mu) + \frac{1}{2} (\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu)$$

$$\text{and wts: } \sigma^\mu \bar{\sigma}^\mu = -\bar{\sigma}^\mu \sigma^\mu$$

$$\text{what about } \sigma^0 \bar{\sigma}^0 = 1 \cdot 1 = \text{Id} = \bar{\sigma}^0 \sigma^0$$

$$\text{we } (\bar{\sigma}^\mu)^{\dot{a}a} = \varepsilon^{ab} \varepsilon^{\dot{a}\dot{b}} (\sigma^\mu)_{b\dot{b}}$$

$$\begin{aligned}
 (\bar{\sigma}^\mu)^{\dot{a}a} (\sigma^\nu)_{a\dot{c}} &= \varepsilon^{ab} \varepsilon^{\dot{a}\dot{b}} (\sigma^\mu)_{b\dot{b}} (\sigma^\nu)_{a\dot{c}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
 &= (\sigma^\mu)_{b\dot{b}} (\sigma^\nu)^{\dot{b}b}_c \\
 &= (\sigma^\mu \sigma^\nu)^{\dot{a}c}
 \end{aligned}$$

$$\begin{aligned}
 (\sigma^\nu)_{c\dot{a}} (\bar{\sigma}^\mu)^{\dot{a}a} &= (\sigma^\nu)_{c\dot{a}} \varepsilon^{ab} \varepsilon^{\dot{a}\dot{b}} (\sigma^\mu)_{b\dot{b}} \\
 &= (\sigma^\nu)^{\dot{b}b}_c (\sigma^\mu)_{b\dot{b}}^a \\
 &= (\sigma^\nu \sigma^\mu)^a_c
 \end{aligned}$$

we know that without the $\perp$, the pauli matrices satisfy

$$[\sigma^a, \sigma^b] = i \varepsilon^{abc} \sigma^c$$

$$\sigma^a \sigma^b = \delta^{ab} + i \varepsilon^{abc} \sigma^c$$

$$\sigma^\mu \sigma^\nu - \sigma^\nu \sigma^\mu = i \varepsilon^{\mu\nu c} \sigma^c$$

cons:

$$\sigma^0 \bar{\sigma}^0 = \bar{\sigma}^0 \sigma^0$$

$$\sigma^i \bar{\sigma}^i = \bar{\sigma}^i \sigma^i$$

$$\sigma^0 \bar{\sigma}^i = \bar{\sigma}^i \sigma^0$$

$$\begin{aligned}
 \text{thj: } \sigma^i \bar{\sigma}^j &= -\sigma^j \bar{\sigma}^i = -(\delta^{ij} + i \varepsilon^{ijk} \sigma^k) \\
 &= -i \varepsilon^{ijk} \sigma^k
 \end{aligned}$$

Ex 4.2 Calculate $[\delta_{\epsilon_1}, \delta_{\epsilon_2}]$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \phi \equiv \delta_{\epsilon_1} \delta_{\epsilon_2} \phi - \delta_{\epsilon_2} \delta_{\epsilon_1} \phi$$

$$= \delta_{\epsilon_1} (\epsilon_2 \psi) - \delta_{\epsilon_2} (\epsilon_1 \psi)$$

$$= \delta_{\epsilon_1} (\epsilon_2^a \psi_a) - \delta_{\epsilon_2} (\epsilon_1^a \psi_a)$$

$$= \epsilon_2^a (-i \sigma_{ab}^{\mu} \epsilon_1^{\dagger b} \partial_{\mu} \phi) - \epsilon_1^a (-i \sigma_{ab}^{\mu} \epsilon_2^{\dagger b} \partial_{\mu} \phi)$$

$$= -i \epsilon_2^a \sigma_{ab}^{\mu} \epsilon_1^{\dagger b} \partial_{\mu} \phi + i \epsilon_1^a \sigma_{ab}^{\mu} \epsilon_2^{\dagger b} \partial_{\mu} \phi$$

$$= i \sigma_{ab}^{\mu} \partial_{\mu} \phi (\epsilon_1^a \epsilon_2^{\dagger b} - \epsilon_2^a \epsilon_1^{\dagger b})$$

$$= i p \cdot \phi \epsilon$$

$\uparrow$
Poincaré generator

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \psi_a = \delta_{\epsilon_1} \delta_{\epsilon_2} \psi_a - \delta_{\epsilon_2} \delta_{\epsilon_1} \psi_a$$

$$= \delta_{\epsilon_1} (-i \sigma_{ab}^{\mu} \epsilon_2^{\dagger b} \partial_{\mu} \psi) - \delta_{\epsilon_2} (-i \sigma_{ab}^{\mu} \epsilon_1^{\dagger b} \partial_{\mu} \psi)$$

$$= -i \sigma_{ab}^{\mu} \epsilon_2^{\dagger b} \partial_{\mu} (\epsilon_1^c \psi_c) + i \sigma_{ab}^{\mu} \epsilon_1^{\dagger b} \partial_{\mu} (\epsilon_2^c \psi_c)$$

$$= i \sigma_{ab}^{\mu} \partial_{\mu} \psi_c (\epsilon_2^c \epsilon_1^{\dagger b} - \epsilon_1^c \epsilon_2^{\dagger b})$$

$$= i p \cdot \psi \epsilon$$

p7/ EH: What's that about $[Q, a(p)] = \hbar b(p)$

$$Q_M = \begin{pmatrix} Q^\dagger a \\ Q^\dagger a^\dagger \end{pmatrix}$$

Hint: think of $Q^\dagger a$ and $a(p)$ both as operators on ket space, so says:

$$\begin{pmatrix} [Q_1, a(p)] \\ [Q_2, a(p)] \end{pmatrix} = \begin{pmatrix} p_1 b(p) \\ p_2 b(p) \end{pmatrix}$$

$$[Q_a, a(p)] |0\rangle$$

$$= [Q_a, a(p)] a^\dagger(k) |0\rangle$$

$$= (Q_a a(p) a^\dagger(k) - a(p) Q_a a^\dagger(k)) |0\rangle$$

$$= \{Q_a [a(p), a^\dagger(k)] + a^\dagger(k) a(p) Q_a - a(p) Q_a a^\dagger(k)\} |0\rangle$$

$\delta(p-k)$ is a c-number

$$= \delta(p-k) \underbrace{Q_a |0\rangle}_0 - a(p) Q_a a^\dagger(k) |0\rangle$$

(Why?) $[Q, a^\dagger] = 0$ by (4.11) p 7/

$$= [\delta(p-k) - a(p) a^\dagger(k)] Q_a |0\rangle$$

$$= a^\dagger a Q_a |0\rangle$$

$$= a_k^\dagger a_p \int d\vec{p} \hbar \omega_p a_p^\dagger b_p - b_p a_p |0\rangle$$

$$a_k^\dagger a_p a_p^\dagger b_p$$

$$a_k^\dagger a_p^\dagger a_p b_p$$

Any-thing
p71 $\delta_\epsilon a_-(p)$

$$\begin{aligned} & \delta_\epsilon \phi = \epsilon \psi \\ & = \delta_\epsilon \int d^3p [a_-(p) e^{ipx} + a_+^\dagger(p) e^{-ipx}] \\ & = \int d^3p [\delta_\epsilon a_-(p) e^{ipx} + \delta_\epsilon a_+^\dagger(p) e^{-ipx}] \\ & = \epsilon \psi \end{aligned}$$

$$= \epsilon \sum_{s=\pm} \int d^3p [b_s(p) \underline{P}_L u_s(p) e^{ipx} + b_s^\dagger(p) \underline{P}_L v_s(p) e^{-ipx}]$$

$\downarrow$ mod ϵ up e^{ipx}

$\downarrow$ mod ϵ up e^{-ipx}

$$\delta_\epsilon a_-(p) = \epsilon \sum_{s=\pm} b_s(p) \underline{P}_L u_s(p)$$

$$\delta_\epsilon a_+^\dagger(p) = \epsilon \sum_{s=\pm} b_s^\dagger(p) \underline{P}_L v_s(p)$$

$\downarrow$

$\langle p | \epsilon \epsilon_s = - (u_+ = \bar{u}_+)$
 $\epsilon \epsilon_s = +$

$$= [\epsilon^\dagger \epsilon_a b_-(p)]$$

$\downarrow$ $|p\rangle_a$ for $s=+$ 0 for $s=-$

$$= \epsilon^a |p\rangle_a b_+^\dagger(p)$$

$\langle \epsilon^\dagger p \rangle ?$

"Using $-|p\rangle\langle p| = p_{ab}$, it is easy to see that $[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \phi(p) = a^\mu p_\mu \phi(p)$ for $\phi(p)$ any one of the 'creation/annihilation operators'."

$$(\delta_{\epsilon_1} \delta_{\epsilon_2} - \delta_{\epsilon_2} \delta_{\epsilon_1}) a_-(p)$$

$$= \delta_{\epsilon_1} ([\epsilon_2 p] b_-(p) - [\epsilon_1 p] b_-(p))$$

$$= [\epsilon_2 p] \langle \epsilon_1 p \rangle a_-(p) - [\epsilon_1 p] \langle \epsilon_2 p \rangle a_-(p)$$

$$= a_-(p) ([\epsilon_1 p] \langle p \epsilon_2 \rangle - [\epsilon_2 p] \langle p \epsilon_1 \rangle)$$

$$= a_-(p) ([\epsilon_2] p_{ab} |\epsilon_1\rangle - [\epsilon_1] p_{ab} |\epsilon_2\rangle)$$

?

$$Q_m = \begin{pmatrix} |Q\rangle a \\ |Q\rangle a^\dagger \end{pmatrix}$$

$$\delta_\epsilon \theta = [\bar{\epsilon}_m Q_m, \theta] = [\epsilon Q + \langle \epsilon Q \rangle, \theta]$$

$$\begin{aligned} \delta_\epsilon \theta &= [\epsilon Q + \langle \epsilon Q \rangle, \theta] \\ &= [\epsilon Q, \theta] + [\langle \epsilon Q \rangle, \theta] \end{aligned}$$

$$\text{let } \theta = a_- \phi$$

$$\begin{aligned} \delta_\epsilon a_- \phi &= [\epsilon p] b_- \phi \\ &= [\epsilon Q, a_- \phi] + [\langle \epsilon Q \rangle, a_- \phi] \\ &= [\epsilon Q] a_- \phi - a_- \phi [\epsilon Q] + \langle \epsilon Q \rangle a_- \phi - a_- \phi \langle \epsilon Q \rangle \end{aligned}$$

$$\delta_\epsilon a_- = [\epsilon Q + \langle \epsilon Q \rangle, a_-] = [\epsilon p] b_-$$

$$\delta_\epsilon b_- = [\quad , b_-] = \langle \epsilon p \rangle a_-$$

$$\delta_\epsilon a_+ = [\quad , a_+] = \langle \epsilon p \rangle b_+$$

$$\delta_\epsilon b_+ = [\quad , b_+] = [\epsilon p] a_+$$

$$[\epsilon Q, a_-] + [\langle \epsilon Q \rangle, a_-] = [\epsilon p] b_- \quad \begin{matrix} b_+ \\ \langle \epsilon Q \rangle a_- - a_- \langle \epsilon Q \rangle \end{matrix}$$

$$[\epsilon Q, b_+] + [\langle \epsilon Q \rangle, b_+] = [\epsilon p] a_+ \quad \begin{matrix} a_- \\ \langle \epsilon Q \rangle b_+ - b_+ \langle \epsilon Q \rangle \end{matrix}$$

$$\sigma = \langle \epsilon, \omega \rangle \quad [[0, a], b] = [0, a] b - b [0, a] = 0ab - a0b - b0a + b00$$

$$[a [0, b]] = a [0, b] - [0, b] a = a0b - a00 - 0ba + 0ab$$

doesn't work

The answer is apparently $[\epsilon p] a_+ b_+^\dagger - [\epsilon p] b_- a_-^\dagger$

$$= [\epsilon Q + \langle \epsilon Q \rangle, b_-] b_+^\dagger - [\epsilon Q + \langle \epsilon Q \rangle, a_-] a_+^\dagger$$

?

$$\{[Q]_a, \langle Q^\dagger | b \rangle\} = \text{Pair} \cdot \text{sum of number operators}$$

$$= [Q]_a \langle Q^\dagger | b \rangle + \langle Q^\dagger | b \rangle [Q]_a$$

$$\int d\tilde{p} [p]_a \langle p | b \rangle (a_+ b_+^\dagger - b_- a_-^\dagger) (a_- b_-^\dagger - b_+ a_+^\dagger) + \int d\tilde{p} d\tilde{q} \langle p | b \rangle [p]_a (a_- b_-^\dagger - b_+ a_+^\dagger) (a_+ b_+^\dagger - b_- a_-^\dagger)$$

$$(a_+ b_+^\dagger - b_- a_-^\dagger) (p) (a_- b_-^\dagger - b_+ a_+^\dagger) (q)$$

$$= a_+ b_+^\dagger a_- b_-^\dagger - a_+ b_+^\dagger b_- a_-^\dagger - b_- a_-^\dagger a_+ b_+^\dagger + b_- a_-^\dagger b_+ a_+^\dagger$$

$$\langle 0 | a_+ a_- b_+^\dagger b_-^\dagger - a_+ a_+^\dagger b_+^\dagger b_-^\dagger - a_-^\dagger a_- b_- b_-^\dagger + a_+^\dagger a_- b_- b_+^\dagger | 0 \rangle$$

$$[a_+, a_+^\dagger] + a_+^\dagger a_+$$

$$[b_-, b_-^\dagger] + b_- b_-^\dagger$$

$$= - [a_+, a_+^\dagger] b_+^\dagger b_+ - [b_-, b_-^\dagger] a_-^\dagger a_-$$

$$= - b_+^\dagger b_+ (p) - a_-^\dagger a_- (p)$$

sum of number operators

4.13 $A_n^{MHV} = A_n [g^- g^- g^+ g^+ \dots g^+] (\eta_1)^4 (\eta_2)^4 + A_n [g^- g^+ g^- g^+ \dots g^+] (\eta_1)^4 (\eta_3)^4 + \text{perms}$
 $+ A_n [g^- \lambda^{123} \lambda^4 g^+ \dots g^+] (\eta_1)^4 (\eta_{21} \eta_{13} \eta_{23} \eta_{34}) + \text{perms}$
 $+ A_n [g^- s^{12} s^{34} g^+ \dots g^+] \eta_{21} \eta_{22} \eta_{33} \eta_{34} + \text{perms}$
 $+ A_n [\lambda^{123} \lambda^4 \lambda^{234} \lambda^1 g^+ \dots g^+] \eta_{11} \eta_{12} \eta_{13} \eta_{24} \eta_{32} \eta_{33} \eta_{34} \eta_{41} + \text{perms}$
 $+ A_n [\lambda^{214} \lambda^3 s^{23} s^{14} g^+ \dots g^+] \eta_{12} \eta_{13} \eta_{14} \eta_{23} \eta_{32} \eta_{33} \eta_{41} \eta_{44} + \text{perms}$
 $+ A_n [s^{12} s^{23} s^{34} s^{41} g^+ \dots g^+] \eta_{12} \eta_{13} \eta_{23} \eta_{33} \eta_{34} \eta_{44} \eta_{41} + \text{perms}$

$\tilde{Q}_4 A_n = 0$

Let $A_n: \tilde{Q}_4 A_n = \sum_{i=1}^n |i\rangle \eta_{i4} A_n$

$= (|1\rangle \eta_{14} + |2\rangle \eta_{24} + \dots + |n\rangle \eta_{n4}) A_n$
 $+ |13\rangle \eta_{34}$

$\tilde{Q}_4 A_n = 0 \Rightarrow$ every independent Grassmann monomial must vanish

E.g. say: for example, let's go with $(\eta_1)^4 (\eta_2)^4 \eta_{34}$ as an independent monomial

clearly, we have: $A_n [g^- g^- g^+ g^+ \dots g^+] (\eta_1)^4 (\eta_2)^4 \cdot (\eta_{34})$ as a term when we hit it with $\tilde{Q}_4$

we want to get: $\eta_{11} \eta_{12} \eta_{13} \eta_{14} \eta_{21} \eta_{22} \eta_{23} \eta_{24} \eta_{34}$

so we need monomials that are missing one of $\{\eta_{14}, \eta_{24}, \eta_{34}, \eta_{44}, \dots, \eta_{n4}\} \in \tilde{Q}_4$

only three matter since η_{44}, η_{54} , etc. are not in $(\eta_1)^4 (\eta_2)^4 \eta_{34}$

The one missing η_{34} we just covered, so the 2 remaining ones:

$\eta_{23} \eta_{12} \eta_{13} \eta_{21} \eta_{22} \eta_{23} \eta_{24} \eta_{34} \in A_n [g^- \lambda^{123} g^+ \lambda^4, g^+ \dots g^+]$

and $\eta_{11} \eta_{12} \eta_{13} \eta_{14} \eta_{21} \eta_{22} \eta_{23} \eta_{34} \in A_n [g^- \lambda^{123} \lambda^4, g^+ \dots g^+]$

Thus, we get:

$A_n [g^- \lambda^{123} g^+ \lambda^4, g^+ \dots g^+] |1\rangle + A_n [g^- \lambda^{123} \lambda^4 g^+ \dots g^+] |2\rangle + A_n [g^- g^- g^+ \dots g^+] |3\rangle = 0$

This is the same as 4.32 up to signs
 (figure out later - also see 'qur' on p 83)

4.14 $\tilde{Q}_A \cdot A_n = 0$
 $Q^A \cdot \delta^{(4)}(\tilde{Q})$
 $= Q^A \cdot \frac{1}{2^4} \prod_{A=1}^4 \tilde{Q}_{Aa} \tilde{Q}^a_A \rightarrow$ can we get this into the form $\langle \tilde{Q}, \tilde{Q} \rangle \sim p^4$?

or directly:

$$\sum_{i=1}^n [k] \frac{\partial}{\partial \eta_{iA}} \cdot \frac{1}{2^4} \prod_{A=1}^4 \sum_{i,j=1}^n \langle ij \rangle \eta_{iA} \eta_{jA}$$

should really do: $\sum_{k=1}^n [k] \frac{\partial}{\partial \eta_{kB}}$ where B is fixed

$$= \sum_{k=1}^n [k] \frac{1}{2^4} \sum_{i,j=1}^n \langle ij \rangle \left(\frac{\partial \eta_{iA}}{\partial \eta_{kB}} \eta_{jA} + \eta_{iA} \frac{\partial \eta_{jA}}{\partial \eta_{kB}} \right)$$

$$= \frac{1}{2^4} \sum_{k=1}^n [k] \sum_{i,j=1}^n \langle ij \rangle (\delta_{ki} \eta_{jB} + \eta_{iB} \delta_{kj})$$

$$= \frac{1}{2^4} \sum_{k=1}^n [k] \left(\sum_j \langle kj \rangle \eta_{jB} + \sum_i \langle ik \rangle \eta_{iB} \right)$$

$$= \frac{1}{2^4} \sum_{kj} [k] \eta_{jB} + \sum_{ik} [k] \eta_{iB}$$

both $\sum_k [k] = 0$ by momentum conservation

Wish property of soft spinors allows you to demonstrate explicitly that $\tilde{Q}_A \delta^{(4)}(\tilde{Q}) = 0$?
 I'll guess that it's Schouten; let's see:

$$\sum_{k=1}^n [k] \eta_{kB} \left(\frac{1}{2^4} \prod_{A=1}^4 \sum_{i,j=1}^n \langle ij \rangle \eta_{iA} \eta_{jA} \right) \quad \text{Schouten: } 1i \langle jk \rangle + 1j \langle ki \rangle + 1k \langle ij \rangle = 0$$

E.g. for $n=4$: $(1i \eta_{iB} + 1j \eta_{jB} + 1k \eta_{kB} + 1l \eta_{lB}) \cdot \left(\sum_{i,j=1}^4 \langle ij \rangle \eta_{i1} \eta_{j1} \eta_{i2} \eta_{j2} \eta_{i3} \eta_{j3} \eta_{i4} \eta_{j4} \right)$

$1i \langle 23 \rangle$
 $1j \langle 32 \rangle$
 $2 \langle 13 \rangle$
 $2 \langle 31 \rangle$
 $3 \langle 12 \rangle$
 $3 \langle 21 \rangle$

problem: the 2 have different coefficients:

$$\eta_{1B} (\eta_2)^4 (\eta_3)^4 \text{ and } \eta_{2B} (\eta_1)^4 (\eta_3)^4$$

what about $\langle 12 \rangle$ terms?
 16 Ap = even - should vanish
 more do not vanish bc
 $\langle 12 \rangle \eta_{11} \eta_{12} \eta_{13} \eta_{14} \eta_{21} \eta_{22} \eta_{23} \eta_{24} + \langle 21 \rangle \eta_{21} \eta_{22} \eta_{23} \eta_{24} \eta_{11} \eta_{12} \eta_{13} \eta_{14}$

Let's consider the non-duplicates for now

$$(1+2+3+4) \left(\langle 12 \rangle + \langle 21 \rangle + \langle 13 \rangle + \langle 31 \rangle + \langle 14 \rangle + \langle 41 \rangle + \langle 23 \rangle + \langle 32 \rangle + \langle 24 \rangle + \langle 42 \rangle + \langle 34 \rangle + \langle 43 \rangle \right)$$

$$1) \cdot (\langle 12 \rangle + \langle 43 \rangle + \langle 44 \rangle + \langle 23 \rangle + \langle 24 \rangle + \langle 34 \rangle)$$

2)
3)
4)

$1) \langle 12 \rangle$	$1) \langle 13 \rangle$	$1) \langle 14 \rangle$	$1) \langle 23 \rangle$	$1) \langle 24 \rangle$	$1) \langle 34 \rangle$
$2) \langle 42 \rangle$	$2) \langle 43 \rangle$	$2) \langle 44 \rangle$	$2) \langle 12 \rangle$	$2) \langle 14 \rangle$	$2) \langle 34 \rangle$
$3) \langle 12 \rangle$	$3) \langle 13 \rangle$	$3) \langle 14 \rangle$	$3) \langle 22 \rangle$	$3) \langle 24 \rangle$	$3) \langle 34 \rangle$
$4) \langle 12 \rangle$	$4) \langle 13 \rangle$	$4) \langle 14 \rangle$	$4) \langle 23 \rangle$	$4) \langle 24 \rangle$	$4) \langle 34 \rangle$

~~There are no pairs~~
a bit we can add zero:

Incorrect boxing: There are all redundant

Pres $1) \langle 12 \rangle = \langle 11 \rangle 2) ?$

$$- 2) \langle 11 \rangle - 1) \langle 21 \rangle$$

we have from the others we eliminated: $1) \langle 21 \rangle$ as well.

$$\text{Then: } 1) \langle 21 \rangle + 1) \langle 12 \rangle \\ = 1) \langle 21 \rangle + 1) \langle 12 \rangle + 2) \langle 11 \rangle$$

$$1) \langle 21 \rangle + 1) \langle 12 \rangle = 0 \text{ by Schur}$$

They cover all cases of $1) \langle ij \rangle$: $(1,1,1)$ are different
 $(1,2,0)$ has $1) \langle ij \rangle$ or $1) \langle ji \rangle$
 $(3,0) = \langle 11 \rangle 1) \langle ij \rangle = 0$ done.

we need to worry about those with η 's:

$$1) \langle 23 \rangle \eta_{15} \eta_{21} \eta_{22} \eta_{23} \eta_{24} \eta_{31} \eta_{32} \eta_{33} \eta_{34} \\ + 2) \langle 31 \rangle \eta_{25} \eta_{31} \eta_{32} \eta_{33} \eta_{34} \eta_{11} \eta_{12} \eta_{13} \eta_{14} \\ + 3) \langle 12 \rangle \eta_{35} \eta_{12} \eta_{13} \eta_{14} \eta_{21} \eta_{22} \eta_{23} \eta_{24}$$

$$\text{If any we had } \eta_{15} \approx \eta_{11} \eta_{12} \eta_{13} \eta_{14}$$

$$(e_1)^{\eta_{15}} (e_1)_{15} \approx (e_1)^{\eta_{11} \eta_{12} \eta_{13} \eta_{14}} (e_1)_{11} (e_1)_{12} (e_1)_{13} (e_1)_{14}$$

Maybe it's not in Dirac if we pull it in:

$$\sum_k \langle k | \eta_{kB} \cdot \frac{1}{2^9} \prod_{A=1}^9 \sum_{j=1}^n \langle j | \eta_{jA} \eta_{jA} \rangle$$

$$= \frac{1}{2^9} \prod_{A=1}^9 \left(\sum_{j=1}^n \langle j | \eta_{jA} \eta_{jA} \rangle \right)$$

so this is zero if it vanishes for each/all fixed A

$$e.g. \langle 1 | \eta_{1B} \eta_{1A} \rangle + 2 \langle 3 | \eta_{3B} \eta_{3A} \rangle + 3 \langle 4 | \eta_{4B} \eta_{4A} \rangle$$

$$\text{This is } (\langle 1 | \eta_{1B} \eta_{1A} \rangle + 2 \langle 3 | \eta_{3B} \eta_{3A} \rangle + 3 \langle 4 | \eta_{4B} \eta_{4A} \rangle) \eta_{1A} \eta_{2A} \eta_{3A} \eta_{4A}$$

But $B=A$ only for one of the $A=1,3,4$

If $B \neq A$ could there be an anticommuting thing where the η 's really then vanish?

Puh! It's a product: $\prod_{A=1}^9 \leftarrow$ so if one of them is zero, the whole expression is zero

QCD (SU(N) gauge group)

$$\text{We know } \text{Tr}(F_a F_b) = \frac{1}{2} f_{abc} f_{abc}$$

$$\text{Tr}([F_a, F_b]) = 0 \Rightarrow [F_a, F_b] = i f_{abc} F_c$$

b/c no central charge?

$$\text{Tr}([F_a, F_b] F_c) = \text{Tr}(i f_{abc} F_c F_c)$$

$$= i f_{abc} \text{Tr}(F_c F_c) = i f_{abc} \frac{1}{2} \delta_{cc} = i f_{abc} \frac{1}{2} N$$

$$\text{Tr}([F_a, F_b] F_c) = \text{Tr}(F_a F_b F_c - F_b F_a F_c)$$

$$= \text{Tr}(F_a F_b F_c) - \text{Tr}(F_b F_a F_c)$$

$$\text{Tr}(F_a F_b F_c) - \text{Tr}(F_c F_a F_b)$$

$$\Rightarrow \text{Tr}(F_b F_a F_c) - \text{Tr}(F_a F_b F_c)$$

by cyclic property of trace

negative! so

f_{abc} is antisymmetric

in b and c as well

$\Rightarrow$ totally antisymmetric

Ex p 130 (6.37) $\frac{\delta^{(8)}(\vec{Q})}{\langle 12 \rangle \langle 34 \rangle} \int d^4 \eta_1 d^4 \eta_2 \frac{\delta^{(8)}(P)}{\langle l_1 \rangle \langle 2l_2 \rangle \langle l_2 l_3 \rangle \langle l_3 l_4 \rangle \langle 4l_1 \rangle \langle l_4 l_2 \rangle}$

$$\delta^{(8)}(P) = -l_2 \cdot \eta_1 + l_3 \cdot \eta_2 + l_4 \cdot \eta_1 + l_1 \cdot \eta_2$$

Ex p 18 p 86 $\delta^{(8)}(\vec{Q}) = \frac{1}{2^4} \prod_{i,j=1}^4 \langle ij \rangle \eta_{iA} \eta_{jA}$

↓ BCFW shift

$$= \frac{2}{2^4} \prod_{A=1}^4 \left(2 \langle 12 \rangle \hat{\eta}_{1A} \eta_{2A} + 2 \sum_{j=3}^4 \langle 1j \rangle \hat{\eta}_{1A} \eta_{jA} + 2 \sum_{j=3}^4 \langle 2j \rangle \eta_{2A} \eta_{jA} + \sum_{j=3}^4 \eta_{1A} \eta_{jA} \right)$$

$$\begin{aligned} & 2 \langle 12 \rangle \hat{\eta}_{1A} \eta_{2A} \\ & 2 \langle 12 \rangle \hat{\eta}_{1A} \eta_{2A} \quad \text{since } \langle 11 \rangle = 0 \\ & = 2 \langle 12 \rangle (\eta_{1A} + z \eta_{2A}) \\ & = 2 \langle 12 \rangle \eta_{1A} \quad \text{since } \eta_{2A} \eta_{2A} = 0 \end{aligned}$$

$$\begin{aligned} & 2 \sum_{j=3}^4 \langle 1j \rangle (\eta_{1A} + z \eta_{2A}) \eta_{jA} + 2 \sum_{j=3}^4 \langle 2j \rangle \eta_{2A} \eta_{jA} \\ & = 2 \sum_{j=3}^4 \langle 1j \rangle \eta_{1A} \eta_{jA} + 2 \sum_{j=3}^4 \langle 1j \rangle z \eta_{2A} \eta_{jA} + 2 \sum_{j=3}^4 \langle 2j \rangle \eta_{2A} \eta_{jA} \\ & \text{does not change} \end{aligned}$$

Ex (a.9.8) p 84 $\delta^{(8)}(P) = \frac{1}{2^4} \prod_{i=1}^4 \left[-\langle l_2 l_3 \rangle \eta_{l_2} \eta_{l_3} - \langle l_2 l_4 \rangle \eta_{l_2} \eta_{l_4} - \langle l_2 l_1 \rangle \eta_{l_2} \eta_{l_1} + \dots - \langle l_1 l_2 \rangle \eta_{l_1} \eta_{l_2} \right]$

$$\text{so } \int d^4 \eta_1 d^4 \eta_2 = \frac{1}{2^4} \frac{d}{d\eta_{l_1}} \dots \frac{d}{d\eta_{l_4}} \frac{1}{d\eta_{l_3}} \frac{d}{d\eta_{l_2}} \frac{2^4 \langle l_1 l_2 \rangle^4 \eta_{l_1} \eta_{l_2} \eta_{l_3} \eta_{l_4}}{2^4} = \langle l_1 l_2 \rangle^4$$

$$\begin{aligned} \text{so } & \frac{\delta^{(8)}(\vec{Q})}{\langle 12 \rangle \langle 34 \rangle} \int d^4 \eta_1 d^4 \eta_2 \frac{\delta^{(8)}(P)}{\langle l_1 \rangle \langle 2l_2 \rangle \langle l_2 l_3 \rangle \langle l_3 l_4 \rangle \langle 4l_1 \rangle \langle l_4 l_2 \rangle} \\ & = \frac{\delta^{(8)}(\vec{Q})}{\langle 12 \rangle \langle 34 \rangle} \frac{\langle l_1 l_2 \rangle^4 (-1)}{\langle l_1 \rangle \langle 2l_1 \rangle \langle l_1 l_2 \rangle \langle l_2 \rangle \langle 4l_1 \rangle \langle l_4 l_2 \rangle} \\ & = - \frac{\delta^{(8)}(\vec{Q}) \langle l_1 l_2 \rangle^2}{\langle 12 \rangle \langle 34 \rangle \langle l_1 \rangle \langle 2l_2 \rangle \langle l_3 \rangle \langle 4l_1 \rangle} \end{aligned}$$

$$A_{4, \text{tree}}[1234] = A_4^{\text{MHV}} = \frac{\delta^{(8)}(\vec{Q})}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \quad \text{(p 84)}$$

$$4.3 \{ |Q\rangle_a, \langle Q^+|_b \}$$

$$= |Q\rangle_a \langle Q^+|_b + \langle Q^+|_b |Q\rangle_a$$

↓

$$\int dp |p\rangle_a (a^\dagger(p) b^\dagger(p) - b(p) a^\dagger(p))$$

$$0 = \langle 0 | \dots$$

$$[Q, Q^3] + [Q^3, Q] = Q^3$$

$$(Q - d[Q^3]) = (Q - d[Q^3])$$

$$\rightarrow -d(\frac{1}{2}d - \frac{1}{2}d + d) q b [Q^3] =$$

$$(\frac{1}{2}d - \frac{1}{2}d + d) - d q b [Q^3] =$$

$$\rightarrow -d(\frac{1}{2}d - \frac{1}{2}d + d) q b \langle Q^3 \rangle +$$

$$(\frac{1}{2}d + d - \frac{1}{2}d - d) - d q b \langle Q^3 \rangle =$$

$$4.19 \langle i | k | j \rangle$$

$$= \langle i | a^\dagger p a b p_b | j \rangle$$

$$= \langle i | a^\dagger p_1 a p_1 b p_2 | j \rangle$$

$$[p, p] = 0 \text{ if massless}$$

$$= [p, p] \langle i p \rangle \langle p j \rangle = \langle i j \rangle \langle p p \rangle = -[p, p] \langle p i \rangle \langle p j \rangle$$

$$\sum_k \langle i | k | j \rangle \langle k | j \rangle ?$$

$$= \langle p j \rangle \langle i p \rangle$$

$$= -\langle p i \rangle \langle p j \rangle = \langle p p \rangle \langle j i \rangle \text{ by Schur's}$$

$$= -[p, p] (\langle p i \rangle \langle p j \rangle + \langle p p \rangle \langle j i \rangle)$$

$$= -[p, p] \langle p p \rangle \langle p j \rangle - [p, p] \langle p p \rangle \langle j i \rangle$$

$$[p, p] \langle p i \rangle \langle p j \rangle$$

$$= [p, p] (\langle i p \rangle \langle j p \rangle - \langle i j \rangle \langle p p \rangle) \text{ by Schur's}$$

can I flip the p's to get another minus?

$$[p, p] \langle i p \rangle \langle p j \rangle + [p, p] \langle p p \rangle \langle j i \rangle = -[p, p] \langle i j \rangle \langle p p \rangle$$

off by a minus sign

$$\text{Wait: } 2 [p, p] \langle i p \rangle \langle p j \rangle = -[p, p] \langle p p \rangle \langle j j \rangle$$

$$-2 p^2 \text{ by (2.24)}$$

$$\langle i | p \cdot p | j \rangle$$

$$= -p^2 \langle i j \rangle$$

Chapter 5



$$(5.12) \quad y_i^{aa} = \frac{|1\rangle \langle 1| [u_{i-1}]^a - |1\rangle \langle 1| [u_i]^a}{\langle 1-1, i \rangle}$$

$$[u_i]^a \equiv \langle 1|_a y_i^{aa} = \langle 1|_a y_{i+1}^{aa}$$

$$\Leftrightarrow X_i^A = \langle 1| \theta_{i,A} \rangle = \langle 1| \theta_{i+1,A} \rangle$$

$$X_{i+1}^A = \langle 1+1| \theta_{i+1,A} \rangle = \langle 1+1| \theta_{i+2,A} \rangle$$

$$X_{i-1}^A = \langle i-1| \theta_{i-1,A} \rangle = \langle i-1| \theta_{i,A} \rangle$$

$$X_i^A - X_{i-1}^A = \langle i| \theta_{i,A} \rangle - \langle i-1| \theta_{i,A} \rangle$$

$$= (\langle i| - \langle i-1|) | \theta_{i,A} \rangle$$

$$\Rightarrow | \theta_{i,A} \rangle = \frac{X_i^A - X_{i-1}^A}{\langle i| - \langle i-1|} \text{Id} = ?$$

Ex 5.1 $\sum_i \langle i|_a \delta_{ij} \delta \langle j|_b \rangle = \epsilon_{ab} \langle j|_b \rangle$

$$A^{ab} - A^{ba} = -A^c_c \epsilon^{ab}$$

$$A^{12} - A^{21} = (A^1_1 - A^2_2) \epsilon^{12} = (A^1_1 - A^2_2) \cdot 1$$

$$\Rightarrow A^{12} - A^{21} + A^{11} + A^{22} = 0$$

if $a=b$: $A^{aa} - A^{aa} = 0$

if $a \neq b$:

$$A^{12} - A^{21} = -A^1_1, -A^2_2 \in A^2_2 = \epsilon_{2a} A^{2a}$$

$$= \epsilon_{21} A^{21} + \epsilon_{22} A^{22}$$

$$= -A^{21}$$

$$A^1_1 = \epsilon_{1a} A^{1a}$$

$$= \epsilon_{11} A^{11} + \epsilon_{12} A^{12}$$

$$= A^{12}$$

$$= -A^{12} - (-A^{21})$$

$$= -A^{12} + A^{21}$$

← mistake: $\epsilon_{12} = -1, \epsilon_{21} = 1 : \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \epsilon_{ab}$

$$\epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Thus, $\sum_i \langle i|_a \partial_{1i} \rangle_b \langle j|_k \rangle$

$$= \sum_i \langle i|_a \frac{\partial}{\partial 1i} \rangle_b \langle j|_c |k\rangle^c$$

$$= \sum_i \langle i|_a \langle j|_c \delta_{ik} \delta_{bi}$$

$$= \underbrace{\langle k|_a \langle j|_b}_{\epsilon_{ac} |k\rangle^c |j|_b}$$

$$\epsilon_{ac} = \frac{A_{ab} - A_{ba}}{-A_c^c}$$

$$\begin{aligned} \langle j|_k \rangle &= \epsilon^{\alpha\beta} j_\alpha |k\rangle_\beta \\ &= \epsilon_{cd} |j\rangle^c |k\rangle^d \end{aligned}$$

so $\sum_i \langle i|_a \frac{\partial}{\partial 1i} \rangle_b (\epsilon_{cd} |j\rangle^c |k\rangle^d)$

$$= \sum_i \langle i|_a \epsilon_{cd} (\delta_{ij} \delta_{bi} |k\rangle^d + \delta_{ik} \delta_{bi} |j\rangle^c)$$

$$= \langle j|_a \epsilon_{ba} |k\rangle^a + \langle k|_a \epsilon_{cb} |j\rangle^c$$

$$= \langle j|_a \epsilon_{ba} |k\rangle^a + \langle k|_a \epsilon_{cb} |j\rangle^c$$

$$= \epsilon_{ba} (\langle j|_a |k\rangle^a - \langle k|_a |j\rangle^c)$$

$$A = \begin{pmatrix} j_1 & k_1 \\ j_2 & k_2 \end{pmatrix}$$

$$\langle j|_a \langle k|_b - \langle k|_a \epsilon_{ba} |j\rangle^c$$

$$= \langle j|_a \langle k|_b - \langle k|_a \langle j|_b$$

$$= A_{ab} - A_{ba}$$

$$= A_c^c \epsilon_{ab}$$

$$= (|j\rangle_2 |k\rangle_1 - |j\rangle_1 |k\rangle_2) \epsilon_{ab}$$

$$= \langle j|_k \rangle \epsilon_{ab}$$

$$A_{ab} - A_{ba} = -A_c^c \epsilon_{ab} ?$$

$$A_c^c = \langle j|_k \rangle = \det A \text{ where } A = \begin{pmatrix} j & k \end{pmatrix}$$

$$A_1^1 + A_2^2 = \epsilon^{1a} A_{a1} + \epsilon^{2a} A_{a2} = A_{21} - A_{12}$$

I don't remember to actually

$$\begin{pmatrix} \langle j_1 | \langle k_1 | \\ \langle j_2 | \langle k_2 | \end{pmatrix}$$

Ex 5.3 $p_{aa} = -i \frac{\partial}{\partial x_{aa}}$ $\int da \delta^3 A = \int da A - da A$

$\int p_{aa} \int x^{bb} = \int p_{aa} \frac{x^{bb}}{x^2} = \int A$

$= -i \int \frac{\partial}{\partial x_{aa}} \left(\frac{x^{bb}}{x^2} \right)$

$= -i \int \left(\frac{\partial}{\partial x_{aa}} \frac{\epsilon^{ab} \epsilon^{ba} x^{cc}}{x^2} \right)$

$= -i \epsilon^{cb} \epsilon^{bi} \left(\frac{\delta_{ac} \delta_{ai}}{x^2} + -2x \delta_{ac} \delta_{ai} \right)$

$= -i \epsilon^{cb} \epsilon^{bi} \left(\frac{1}{x^2} - 2x^2 \right) \delta_{ac} \delta_{ai}$

vs $k^{aa} x^{bb}$

$= -i x^{ac} x^{ia} \frac{\partial}{\partial x^{cc}} x^{bb}$

$= -i x^{ac} x^{ia} \delta^{cb} \delta^{eb}$

$\begin{pmatrix} 1 & 1 \\ 5 & 5 \end{pmatrix} = A$

$\begin{pmatrix} 1 & 1 \\ 5 & 5 \end{pmatrix} = A$

$(\langle j| \dots \rangle + \langle j| \dots \rangle) \dots$

$\langle j| \dots \rangle + \langle j| \dots \rangle$

$\langle j| \dots \rangle + \langle j| \dots \rangle$

$(\langle j| \dots \rangle - \langle j| \dots \rangle)$

$\langle j| \dots \rangle = \dots$

$\langle j| \dots \rangle - \langle j| \dots \rangle = \dots$

$\dots A = \dots$

$\dots A = \dots$

$\dots = \dots$

$\dots = \dots$

Supernumeration Twistos p142, 107

$$\bar{Z}_i \bar{A} \equiv (|i\rangle, [M_i] |X_A), \bar{A} = (a, A)$$

$$X_{iA} = \langle i | \theta_{iA} \rangle = \langle i | \theta_{i+1,A} \rangle, A=1,2,3,4$$

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$$\Rightarrow \langle i | (\theta_{iA} - \theta_{i+1,A}) = \langle i | \theta_{iA} \rangle$$

$$\langle i | \theta_{iA} - \langle i | \theta_{i+1,A} = \langle i | \theta_{iA}$$

$$\Rightarrow \langle i | \theta_{iA} = \langle i | \theta_{i+1,A}$$

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p107

$$(|i\rangle^b \langle i-1|_a \langle i-1|_b \langle i|_a) y_i^{aa} y_i^{aa}$$

$$|i\rangle^b \langle i-1|_a \sigma_{\mu}^{aa} = |i\rangle^b \langle i-1|_a \varepsilon_{ca} \sigma_{\mu}^{aa} = \varepsilon^{ab} \varepsilon^{ab} (\sigma_{\mu})_{bb}$$

$$|i\rangle^b \langle i-1|_a \varepsilon^{ac} \varepsilon^{ac} (\sigma_{\mu})_{cc}$$

$$(|i\rangle^b \langle i-1|_a \varepsilon^{ac} (\sigma_{\mu})_{cc}$$

$$|i\rangle^b \langle i-1|_a \varepsilon^{ac} \varepsilon^{ac} (\sigma_{\mu})_{cc}$$

$$|i\rangle^b \langle i-1|_a \varepsilon_{da} \varepsilon^{ac} \varepsilon^{ac} (\sigma_{\mu})_{cc}$$

$$\text{Wd: } |i\rangle^b y_i^{aa} \Rightarrow |i\rangle^a y_i^{ba}$$

$$|i\rangle_c \varepsilon^{cb} y_i^{aa}$$

$$= |i\rangle_c \varepsilon^{cb} \varepsilon^{ad} y_i^{da}$$

$$= |i\rangle_c \varepsilon^{cb} \varepsilon^{ad} y_i^{da}$$

$$= |i\rangle_c \varepsilon^{ac} \varepsilon^{bd} y_i^{da}$$

$$= |i\rangle^a y_i^{ba}$$

Ex 5.8) $|m_i]_a = -(y_i)_{ab} |i]^b$

$|m_i]_a \equiv \langle i | a y_i^a$

$|m_i]_b \equiv \langle i | a y_i^a$

$|m_i]_a \stackrel{?}{=} \epsilon_{ab} |m_i]_b = \epsilon_{ab} \langle i | a (y_i)^{ab}$

$\stackrel{?}{=} \epsilon_{ab} \langle i | a y_i^m \bar{\sigma}_m^{ab}$

$\stackrel{?}{=} \epsilon_{ab} \epsilon_{ab} \langle i | a y_i^m \bar{\sigma}_m^{ab}$

$\stackrel{?}{=} -(\sigma_m)_{ab} \langle i | a y_i^m = \langle i | a y_i^m = - (y_i)_{ab} |i]^b$

Try Ex 45) again:

$|i]^b \langle i-1 | a y_i^a$
 $= |i]^b \langle i-1 | a (y_i^m) (\bar{\sigma}_m)^{aa}$

$= \epsilon^{ba} \langle i | c \epsilon_{aa} |i-1]^a (y_i^m) \epsilon^{ae} \epsilon_{ae} (\sigma_m)_{ee}$

$= \epsilon^{ba} \langle i | c \epsilon_{aa} |i-1]^a (y_i^m) \epsilon^{ae} (\sigma_m)_{ee}$
 $= \epsilon^{ba} \langle i | c \epsilon_{aa} |i-1]^a (y_i^m) \epsilon^{ae} (\sigma_m)_{ee}$

$= \epsilon^{ba} \langle i | c \epsilon_{aa} |i-1]^a (y_i^m) \epsilon^{ae} (\sigma_m)_{ee}$ Same pattern as before

${}^{ss}A_{ss} + {}^{ls}A_{ss} =$

${}^{ls}A_{ss} =$

${}^{ss}A_{ss} = {}^{ss}A$

${}^{ss}A_{ss} = {}^{ss}A$

${}^{ss}A_{ss} =$

$\text{Ans } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} : \text{Ans } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \text{Ans } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} x_2 & x_3 & x_4 & x_5 \\ y_2 & y_3 & y_4 & y_5 \\ z_2 & z_3 & z_4 & z_5 \\ w_2 & w_3 & w_4 & w_5 \end{pmatrix} \begin{pmatrix} c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} = \begin{pmatrix} a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} \rightarrow \text{Matthematik Sept 18, 2025}$$

Geometrische

$$p_i = y_i^L - y_i^M$$

$$p_{12}^{(1)} = y_1^u - y_{N+1}^u$$

$$\langle 1_a | 1_b \rangle = \langle 1_a | (y_1^{a,b} - y_{1+1}^{a,b}) \rangle := \langle 1_a |$$

$\vec{z}_i = (i; \text{Cen})$ point corresponds to part in main task
 not be related by 2 part $\nwarrow$ null line
 part in main task $\longleftrightarrow$ null line in dual space
 part in main task $\longrightarrow$ part in dual space

$$[\mu_1]^a = \langle i_1 y_1^a \rangle \quad [\mu_{-1}]^a = \langle -1 y_1^a \rangle$$

$$\begin{aligned} |1\rangle^b [u_1]^a - |1\rangle^b [u_1]^a &= |1\rangle^b \langle 1|_a y_1^{aa} - |1\rangle^b \langle 1|_a y_1^{aa} \\ &= (|1\rangle^b \langle 1|_a - |1\rangle^b \langle 1|_a) y_1^{aa} \end{aligned}$$

$$\begin{aligned}\langle f | f \rangle &= \langle f | a | f \rangle^a \\ &= \sum_{a,c} \langle f | a | f \rangle^a \\ &= \sum_{a,c} \langle f | a | f \rangle^a\end{aligned}$$

$$\mu_1 = \lambda_1 y_1 + \lambda_2 y_2$$

$$\mu_{i-1} = \lambda_{i-1} y_{i-1} = \lambda_{i-1} y_i$$

$$\mu_1 - \mu_{n-1} = (\lambda_1 - \lambda_{n-1}) y_i$$

$$\langle 11 | i \rangle = \epsilon_{ac} |1\rangle^c = |1\rangle^a$$

$$\epsilon_{ac} (|1\rangle^b |1\rangle^c - |1\rangle^c |1\rangle^b) (y_i)^a$$

$$\epsilon^{bc} (\langle 1 |_c |1\rangle^c - \langle 1 |_c |1\rangle^c) (y_i)^a$$

$$(2 \langle 1 |_i |1\rangle^i) (y_i)^{ba}$$

$$x^b y^a$$

$$\begin{aligned} & \varepsilon^{bc} \left(\langle 1 | \cdot | a \rangle - \langle 1 | \cdot | a \rangle \right) y_1^{aa} \\ & \varepsilon_{aa} \varepsilon^{bc} \left(\langle 1 | \cdot | a \rangle - \langle 1 | \cdot | a \rangle \right) y_1^{aa} \\ & \varepsilon^{bc} \left(\langle 1 | \cdot | a \rangle - \langle 1 | \cdot | a \rangle \right) y_1^{aa} \end{aligned}$$

$$y_i^a = \varepsilon^{ax} y_x^a$$

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[illegible]

Ques: get $G = \frac{\langle 1,1 | \rangle [1,1] + \langle 1,1 | \rangle [1,1] + \langle 1,1 | \rangle [1,1]}{\langle 1,1 | \rangle \langle 1,1 | \rangle}$

$$P_i = \frac{y_i^u - y_{i+1}^u}{\langle i, i+1 \rangle} - \frac{\langle i, i+1 \rangle \langle i, i+1 \rangle}{\langle i, i+1 \rangle}$$

$$= \frac{|i\rangle \langle i, j+1| [u_{j+1}] - |j+1\rangle \langle i, j+1| [u_j] - |i\rangle \langle j+1, j| [u_j] + |j+1\rangle \langle j+1, j| [u_{j+1}]}{\langle j+1, j| \langle i, j+1|}$$

$$= \mathbb{E}_{\mathbf{X}} [17\langle 1, 1 \rangle] U_{n+1} - (11\langle 1, 1 \rangle \mathbb{E}_{\mathbf{X}} [17\langle 1, 1 \rangle] U_{n+1} + 17\langle 1, 1 \rangle U_{n+1})$$

$$= \frac{|1\rangle\langle 1, 1\rangle C_{1111} - (-|1\rangle\langle 1, 1\rangle C_{11} + |1\rangle\langle 1, 1\rangle C_{1111})}{\langle 1, 1|7\rangle\langle 1, 1\rangle}$$

$$p_i = \frac{1}{2} \left(\langle 1,1 | \psi \rangle \langle 1,1 | + \langle 1,1 | \psi \rangle \langle 1,1 | + \langle 1,1 | \psi \rangle \langle 1,1 | + \langle 1,1 | \psi \rangle \langle 1,1 | \right)$$

Ex 5.12 $\chi_i^A = \langle 10_{1A} \rangle = \langle 10_{1B}, A \rangle \quad |0_{1A}\rangle^A$

$$\begin{aligned} |0_{1A}\rangle - |0_{1A}\rangle &= |1\rangle / \eta_{1A} \\ |0_{1A}\rangle - |0_{1A}\rangle &= |1\rangle / \eta_{1A} \\ \rightarrow \chi_{1A}^A &= \langle 1 | \theta_{1A} \rangle = \langle 1 | \theta_{1A} \rangle \end{aligned}$$

$$\begin{aligned} \chi_{1-1}^A |1\rangle - \chi_1^A |1\rangle &= |1\rangle \langle 1-1 | \theta_{1A} \rangle = |1\rangle \langle 1-1 | \theta_{1A} \rangle \\ &= (|1\rangle \langle 1-1 | - |1-1\rangle \langle 1|) \theta_{1A} \end{aligned}$$

$$\Rightarrow \theta_{n+1} = \frac{x_{n+1,i} - x_{n+1,j}}{t_{n+1,i} - t_{n+1,j}}$$

$$\begin{aligned}
 |1\rangle\eta_{1A} &= |0_{1A}\rangle - |0_{1+1A}\rangle = \frac{X_{1,1}|1\rangle - X_{1,1}|1\rangle}{\langle 1,1|1\rangle} - \frac{X_{1,1}|1+1\rangle - X_{1+1}|1\rangle}{\langle 1,1+1\rangle} \\
 &= \frac{X_{1,1}|1\rangle\langle 1,1+1| - X_{1,1}|1+1\rangle\langle 1,1+1| - X_{1,1}|1+1\rangle\langle 1,1| + X_{1+1}|1\rangle\langle 1,1|}{\langle 1,1|1\rangle\langle 1,1+1\rangle}
 \end{aligned}$$

$$= \frac{\chi_{H1} | \psi_{1, H1} \rangle = (| \psi_{1, H1} \rangle + | \psi_{1, H2} \rangle) \chi_1 + \chi_{H2} | \psi_{2, H1} \rangle}{\langle \psi_{1, H1} | \psi_{1, H1} \rangle}$$

$$= |i\rangle \langle i, i\rangle \chi_{ii} - (-|i\rangle \langle i, i\rangle) \chi_{ii} + |i\rangle \langle i, i\rangle \chi_{ii}$$

$$= \frac{1}{\langle 1,1 \rangle \langle 1,1 \rangle} (\langle 1,1 \rangle \langle 1,1 \rangle \chi_{1,1} + \langle 1,1 \rangle \langle 1,1 \rangle \chi_{1,1} + \langle 1,1 \rangle \langle 1,1 \rangle \chi_{1,1})$$

~~$p_j = y_j - y_{j+1}$~~ $p_i = y_i - y_{i+1}$
 $y_{j,j+1}$ Can we use $y_j = y_{1,j}$ so $y_j - y_{j+1} = y_{1,j} - y_{j+1} = (p_1 + \dots + p_{j-1}) - (p_1 + \dots + p_j) = -p_j$?

$$\langle K | (p_k + \dots + p_{j-1}) (p_j + \dots + p_{r-1}) | r \rangle$$

$$p_j = y_j - y_{j+1}$$

$$y_j = p_j + y_{j+1}$$

$$\text{Define } y_{1j} = p_1 + \dots + p_{j-1}$$

Then we can also define $y_j = y_{1j}$ so that

$$y_j - y_{j+1} = y_{1j} - y_{j+1} = (p_1 + \dots + p_{j-1}) - (p_1 + \dots + p_j) = -p_j$$

Ex 5.11 WTS: $\langle k, j-1, j, r \rangle = \langle j-1, j \rangle \langle k | y_{kj} y_{jr} | r \rangle$

By 5.44:

$$\langle k, j-1, j, r \rangle = \langle k, j-1 \rangle [u_j, m_r] + \langle k, j \rangle [u_r, m_{j-1}] + \langle k, r \rangle [m_{j-1}, m_j] \\ + \langle j, r \rangle [u_k, m_{j-1}] + \langle j, j \rangle [u_k, m_j] + \langle j-1, j \rangle [u_k, m_r]$$

$$= \langle k, j-1 \rangle \langle j | y_j y_r r \rangle + \langle k, j \rangle \langle r y_r y_{j-1} j \rangle + \langle k, r \rangle \langle j-1 y_j y_{j-1} j \rangle \\ + \langle j, r \rangle \langle k y_k y_{j-1} j \rangle + \langle j, j \rangle \langle k y_k y_j j \rangle + \langle j-1, j \rangle \langle k y_k y_r r \rangle$$

Substituting: $\langle r | \rangle \langle j | \rangle + \langle r | \rangle \langle k | \rangle + \langle k | \rangle \langle j | \rangle = 0$

we get: $\langle j-1, j \rangle$

$$\Rightarrow (-\langle k y_j y_r r \rangle \langle j-1, j \rangle + \langle r y_k y_j k \rangle \langle j-1, j \rangle) - \langle k r \rangle \langle j-1 y_j y_j j \rangle = \langle j-1, j \rangle \langle k y_k y_r r \rangle$$

$$-\langle k r \rangle \langle j-1 y_j y_j j \rangle = \langle j-1, j \rangle \langle k y_k y_r r \rangle$$

$$= \langle j-1, j \rangle \langle k y_j y_r r \rangle - \langle j-1, j \rangle \langle r y_j y_k k \rangle - \langle j-1, j \rangle \langle k y_k y_r r \rangle - \langle k r \rangle \langle j-1 y_j y_j j \rangle$$

WANT: $\langle k | y_{kj} y_{jr} \rangle = \langle k | (p_k + \dots + p_{j-1}) (p_j + \dots + p_{r-1}) | r \rangle$

If we could make this $-\langle k y_j y_j r \rangle \langle j-1, j \rangle$

$$0 = \langle k y_j y_j r \rangle \langle j-1, j \rangle + \langle k y_j y_j r \rangle \langle j-1, j \rangle + \langle k y_j y_j r \rangle \langle j-1, j \rangle$$

$$y_{1j} y_{jr} - y_{rk} y_{jk} = (p_j + \dots + p_{j-1}) (y_j - y_{j+1}) - (y_r - y_{r+1}) (y_j - y_{j+1})$$

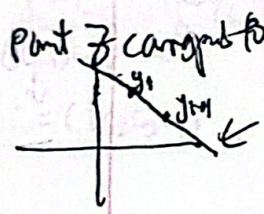
$$y_{kj} y_{jr} = (y_k + \dots + y_{j-1}) (y_j + \dots + y_{r-1})$$

$$(y_k - y_j) (y_j - y_r)$$

$$y_k y_j - y_j y_j - y_k y_r + y_j y_r$$

almost

$$(y_k y_j - y_j y_j - y_k y_r + y_j y_r) = (y_k y_j - y_j y_j - y_k y_r + y_j y_r)$$



Point z corresponds to line defined by y_i, y_{i+1}

$$r y_i + (1-r) y_{i+1} = \tilde{y}_i$$

$$s y_i + (1-s) y_{i+1} = \tilde{y}_{i+1}$$

$$\langle 1 | \tilde{y}_i = r \langle 1 | y_i + (1-r) \langle 1 | y_{i+1}$$

$$= r \langle 1 | + (1-r) \langle 1 |$$

$$= \langle 1 |$$

orthonormal basis

$$K_M \left(\frac{p^M}{p \cdot k} - \frac{p^k}{p \cdot k} \right) = \frac{K_M p^M}{p \cdot k} - \frac{p^k}{p \cdot k} = 1 - 1 = 0$$

$$\frac{\int d\Omega_k \frac{2 p \cdot p}{(p \cdot k)(p \cdot k)}}{\frac{\int d\Omega_k}{4\pi}} = \frac{\int_0^1 dx \int_0^1 dy \delta(\omega_2 - 1)}{[x p \cdot k + y p \cdot k]}$$

$$\frac{x^2 (p \cdot k)^2 + 2xy (p \cdot k)(p \cdot k) + y^2 (p \cdot k)^2}{p \cdot k}$$

WTS: $|1\rangle = \langle 1 | -1, 1 \rangle |1\rangle + \langle 1 | 1, -1 \rangle |1\rangle + \langle 1 | 1, 1 \rangle |1\rangle$

by using momentum conservation

$$\sum_{i=1}^n |1\rangle \langle 1| = 0$$

$$\epsilon^{ab} \epsilon_{bc} = \delta^a_c$$

$$\epsilon^{ab} \epsilon_{bc} = \epsilon^{a1} \epsilon_{1c} + \epsilon^{a2} \epsilon_{2c}$$

$$\epsilon^{a1} \epsilon_{1c} = \delta^a_c - \epsilon^{a2} \epsilon_{2c}$$

$$\epsilon^{a2} \epsilon_{2c} = \delta^a_c - \epsilon^{a1} \epsilon_{1c}$$

$$\epsilon^{a1} \epsilon_{1c} + \epsilon^{a2} \epsilon_{2c} = 2\delta^a_c$$

$$\epsilon^{a1} \epsilon_{1c} = \delta^a_c - \epsilon^{a2} \epsilon_{2c}$$

$$\epsilon^{a2} \epsilon_{2c} = \delta^a_c - \epsilon^{a1} \epsilon_{1c}$$

$$\epsilon^{a1} \epsilon_{1c} = \delta^a_c - \epsilon^{a2} \epsilon_{2c}$$

$$\epsilon^{a2} \epsilon_{2c} = \delta^a_c - \epsilon^{a1} \epsilon_{1c}$$

$$p_i = y_i - y_{i+1}$$

$$p_{i+1} = y_{i+1} - y_{i+2}$$

$$0 = \langle 1 | y_i - \langle 1 | y_{i+1} = \langle 1 | y_i - \langle 1 | y_{i+1}$$

$$[u_i] = \langle 1 | y_i = \langle 1 | y_{i+1}$$

$$[u_{i+1}] = \langle 1 | y_{i+1} = \langle 1 | y_{i+2}$$

$$[u_i] = \langle 1 | y_i = \langle 1 | y_{i+1}$$

$$[u_{i+1}] = \langle 1 | y_{i+1} = \langle 1 | y_{i+2}$$

$$\langle 1 | 1 \rangle = \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

$$= \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

$$= \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

$$= \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

$$= \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

$$\langle 1 | 1 \rangle = \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

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$$\langle 1 | 1 \rangle = \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

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$$= \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

$$= \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

$$= \langle 1 | 1 \rangle^2 - \langle 1 | 2 \rangle^2$$

$$y^a = (y_0, y_1, y_2, y_3)$$

$$y^a = (y_0, y_1, y_2, y_3)$$

$$y^a = (y_0, y_1, y_2, y_3)$$

$$y^a = (y_0, y_1, y_2, y_3)$$

$$y^a = (y_0, y_1, y_2, y_3)$$

$$|1\rangle^b = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \epsilon_{11} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \quad y^a = \begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix} = y^a \epsilon_a$$

$$\epsilon_{66} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\langle 1 | 1 \rangle = \epsilon_{6a} |1\rangle^a = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

$$\langle 1 | 1 \rangle = \det \begin{pmatrix} -1 & 3 \\ 2 & 5 \end{pmatrix} = -5 - 6 = -11$$

$$\text{so } \langle 1 | 1 \rangle y^a = \begin{pmatrix} -110 & -220 \\ -330 & -440 \end{pmatrix}$$

$$\text{Now let's suppose } y^a = \begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix}$$

$$|1\rangle^b = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \quad \epsilon_{11} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\text{so } (|1\rangle^b \langle 1 |^a - |1\rangle^b \langle 1 |^a) = \begin{pmatrix} a=1, b=1 & a=2, b=2 \\ a=2, b=1 & a=1, b=2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -20 \\ -6 & 1 \end{pmatrix}$$

$$\text{so } \langle 1 | 1 \rangle^b = 1, a:$$

$$(-1)y^1_a - 6y^2_a$$

$$b=2, a: (-20)y^1_a + y^2_a$$

$$\text{so for } \langle 1 | 1 \rangle^b y^a = \langle 1 | 1 \rangle^b \cdot \begin{pmatrix} y_0 - y_3 & -y_1 + y_2 \\ -y_1 - y_2 & y_0 + y_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_0 - y_3 & -y_1 + y_2 \\ -y_1 - y_2 & y_0 + y_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_0 - y_3 & -y_1 + y_2 \\ -y_1 - y_2 & y_0 + y_3 \end{pmatrix} = \begin{pmatrix} (1+1)(y_0 - y_3) & (1-1)(-y_1 + y_2) \\ (1+1)(-y_1 - y_2) & (1+1)(y_0 + y_3) \end{pmatrix}$$

Should equal 0

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_0 - y_3 & -y_1 + y_2 \\ -y_1 - y_2 & y_0 + y_3 \end{pmatrix} = \begin{pmatrix} 2(y_0 - y_3) & 0 \\ 0 & 2(y_0 + y_3) \end{pmatrix}$$

More generally,

$$|1\rangle^a \langle 1 |^b = \epsilon_{11} \epsilon_{11} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$= \epsilon_{11} \epsilon_{11} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$= 0 \text{ if } b=1, \text{ then } c=2: (|1\rangle^b \langle 1 |^c - |1\rangle^b \langle 1 |^c) \epsilon_{11} y^a \text{ so } a=1, \epsilon_{11} = 1$$

$$\Rightarrow |1\rangle^b \langle 1 |^c = 2: (|1\rangle^b \langle 1 |^c - |1\rangle^b \langle 1 |^c) \epsilon_{11} y^a \text{ so } a=1, \epsilon_{11} = 1$$

$$\Rightarrow |1\rangle^b \langle 1 |^c = 1: (|1\rangle^b \langle 1 |^c - |1\rangle^b \langle 1 |^c) \epsilon_{11} y^a \text{ so } a=1, \epsilon_{11} = -1$$

$$\Rightarrow |1\rangle^b \langle 1 |^c = 1: (|1\rangle^b \langle 1 |^c - |1\rangle^b \langle 1 |^c) \epsilon_{11} y^a \text{ so } a=1, \epsilon_{11} = -1$$

$$= (|1\rangle^b \langle 1 |^c - |1\rangle^b \langle 1 |^c) \epsilon_{11} y^a = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} y^a$$

so this expression equals

Alternative way to solve:

Minimal way
 $y_{\lambda} = \mu$

$$\langle 1, i+1 | y_{\lambda} = \mu_{i+1}$$

$$\langle 1 | y_{\lambda} = \mu_1$$

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} \lambda_{i+1}^1 \\ \lambda_{i+1}^2 \end{pmatrix} = \begin{pmatrix} \mu_{i+1}^1 \\ \mu_{i+1}^2 \end{pmatrix}$$

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} \lambda_1^1 \\ \lambda_1^2 \end{pmatrix} = \begin{pmatrix} \mu_1^1 \\ \mu_1^2 \end{pmatrix}$$

$$y_{11} \lambda_{i+1}^1 + y_{12} \lambda_{i+1}^2 = \mu_{i+1}^1$$

$$y_{21} \lambda_{i+1}^1 + y_{22} \lambda_{i+1}^2 = \mu_{i+1}^2$$

$$y_{11} \lambda_1^1 + y_{12} \lambda_1^2 = \mu_1^1$$

$$y_{21} \lambda_1^1 + y_{22} \lambda_1^2 = \mu_1^2$$

$$\rightarrow -\frac{\lambda_{i+1}^1}{\lambda_1^1} (y_{11} \lambda_1^1 + y_{12} \lambda_1^2 = \mu_1^1) = -\frac{\lambda_{i+1}^1}{\lambda_1^1} y_{11} - \frac{\lambda_{i+1}^1}{\lambda_1^1} y_{12} = -\frac{\lambda_{i+1}^1}{\lambda_1^1} \mu_1^1$$

$$y_{12} (\lambda_{i+1}^2 - \lambda_{i+1}^1 \frac{\lambda_1^2}{\lambda_1^1}) = \mu_{i+1}^1 - \frac{\lambda_{i+1}^1}{\lambda_1^1} \mu_1^1$$

$$y_{12} \frac{\lambda_1^1 \lambda_{i+1}^2 - \lambda_{i+1}^1 \lambda_1^2}{\lambda_1^1} = \frac{\lambda_1^1 \mu_{i+1}^1 - \mu_1^1 \lambda_{i+1}^1}{\lambda_1^1}$$

$$y_{12} = \frac{\lambda_1^1 \mu_{i+1}^1 - \mu_1^1 \lambda_{i+1}^1}{\lambda_1^1 \lambda_{i+1}^2 - \lambda_{i+1}^1 \lambda_1^2}$$

the same relation in time for
 y_{21}, y_{22} (instead of y_{11}, y_{12}),
 just with μ^2 (instead of μ^1):

$$y_{22} = \frac{\lambda_1^1 \mu_{i+1}^2 - \mu_1^2 \lambda_{i+1}^1}{\lambda_1^1 \lambda_{i+1}^2 - \lambda_{i+1}^1 \lambda_1^2}$$

$$\text{Thus, } y_{a2} = \frac{\lambda_1^a \mu_{i+1}^2 - \mu_1^2 \lambda_{i+1}^a}{\langle 1, i+1 \rangle}$$

$$\text{For } y_{11} = \frac{\mu_{i+1}^1 - y_{12} \lambda_{i+1}^2}{\lambda_{i+1}^1} = \mu_{i+1}^1 - \lambda_{i+1}^2 \left(\frac{\lambda_1^1 \mu_{i+1}^1 - \mu_1^1 \lambda_{i+1}^1}{\langle 1, i+1 \rangle} \right)$$

$$= \mu_{i+1}^1 \langle 1, i+1 \rangle - \lambda_{i+1}^2 \lambda_1^1 \mu_{i+1}^1 + \mu_1^1 \lambda_{i+1}^2 \lambda_{i+1}^1$$

$$= \frac{\mu_{i+1}^1 \lambda_1^1 \lambda_{i+1}^2 - \mu_{i+1}^1 \lambda_{i+1}^2 \lambda_1^1 - \lambda_{i+1}^2 \lambda_1^1 \mu_{i+1}^1 + \mu_1^1 \lambda_{i+1}^2 \lambda_{i+1}^1}{\lambda_{i+1}^1 \langle 1, i+1 \rangle}$$

$$= \frac{\mu_1^1 \lambda_{i+1}^2 - \mu_{i+1}^1 \lambda_1^2}{\langle 1, i+1 \rangle} = y_{11}$$

I should check that I get the same
 result with $y_{11} \lambda_1^1 + y_{12} \lambda_1^2 = \mu_1^1$ (but it's getting late)

$$\text{So, } y_{21} = \frac{\mu_1^2 - \lambda_1^2 y_{22}}{\lambda_1^1} = \frac{\mu_1^2 - \lambda_1^2 \left(\frac{\lambda_1^1 \mu_{i+1}^2 - \mu_1^2 \lambda_{i+1}^1}{\langle 1, i+1 \rangle} \right)}{\lambda_1^1}$$

$$= \frac{\mu_1^2 \lambda_1^1 \langle 1, i+1 \rangle - \lambda_1^2 \lambda_1^1 \mu_{i+1}^2 + \lambda_1^2 \mu_1^2 \lambda_{i+1}^1}{\lambda_1^1 \langle 1, i+1 \rangle}$$

$$= \frac{\mu_1^2 \lambda_{i+1}^2 - \mu_{i+1}^2 \lambda_1^2}{\langle 1, i+1 \rangle} = y_{21}$$

$$\text{Thus, } y_{a1} = \frac{\mu_1^a \lambda_{i+1}^2 - \mu_{i+1}^a \lambda_1^2}{\langle 1, i+1 \rangle}$$

$$\text{Finally, } y_{ab} = \frac{\epsilon^{bc} (\mu_1^a \lambda_{i+1}^c - \mu_{i+1}^a \lambda_1^c)}{\langle 1, i+1 \rangle} \quad \text{there might be a minus sign}$$

Now let's go back to the problem on p. 192:

$$\text{we: } \langle i | = \langle i+1, i \rangle \langle u_{i+1} | + \langle i, i+1 \rangle \langle u_{i+1} | + \dots + \langle i-1, i \rangle \langle u_i |$$

by math exercise:

$$\sum_i |i\rangle \langle i| = 0$$

$$\sum p_i = 0$$

$$\sum p_i = \sum y_i - y_{i+1} + \dots$$

$$= y_1 - y_2 + y_2 - y_3 + y_3 - y_4 + \dots = 0$$

$$\dots + \frac{12 \langle u_1 | - 12 \rangle \langle u_2 |}{\langle 12 \rangle} - \frac{13 \langle u_2 | - 12 \rangle \langle u_3 |}{\langle 23 \rangle} + \dots$$

Wang-Schroeder

$$\text{We use: } y_i = \frac{12^i \langle u_{i-1} |^2 - 12 \rangle \langle u_i |^2}{\langle i-1, i \rangle}$$

Now, back to Blythley, Lan, Speyer, etc.

Suppose we want to find the matrix Q mapping from matrix space to matrix space. We really only need to transfer the K matrix.

$$|2\rangle = \langle 3, 2 \rangle \langle u_1 | + \langle 2, 3 \rangle \langle u_3 | + \langle 1, 3 \rangle \langle u_2 | = -\frac{1}{\langle 1, 2 \rangle} \langle u_1 | - \frac{1}{\langle 2, 3 \rangle} \langle u_3 | + \frac{\langle 1, 3 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} \langle u_2 |$$

$$\begin{pmatrix} \langle 11 \rangle \\ \langle 21 \rangle \\ \langle 31 \rangle \end{pmatrix} = \begin{pmatrix} \frac{\langle 1, 2 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & 0 & \dots & 0 & -\frac{1}{\langle 1, 2 \rangle} \\ \frac{\langle 1, 3 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} & 0 & 0 & 0 \\ 0 & -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 2, 4 \rangle}{\langle 2, 3 \rangle \langle 3, 4 \rangle} & \frac{1}{\langle 3, 4 \rangle} & 0 \end{pmatrix} \begin{pmatrix} a \cdot \langle u_1 | \\ b \cdot \langle u_2 | \\ c \cdot \langle u_3 | \end{pmatrix} \leftarrow \langle u_i | \text{ is basis}$$

If there are 3 particles:

$$\det \begin{pmatrix} \frac{\langle 1, 2 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} & 0 & \frac{1}{\langle 3, 4 \rangle} \\ -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 1, 3 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} & 0 \\ 0 & -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 2, 4 \rangle}{\langle 2, 3 \rangle \langle 3, 4 \rangle} & \frac{1}{\langle 3, 4 \rangle} \\ -\frac{1}{\langle 3, 4 \rangle} & 0 & -\frac{1}{\langle 3, 4 \rangle} & \frac{\langle 3, 1 \rangle}{\langle 3, 4 \rangle \langle 4, 1 \rangle} \end{pmatrix} = Q_4$$

$$3 \text{ particles: } \begin{pmatrix} \frac{\langle 1, 2 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} & \frac{\langle 2, 1 \rangle}{\langle 2, 3 \rangle \langle 3, 1 \rangle} \\ -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 1, 3 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} \\ 0 & -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 2, 1 \rangle}{\langle 2, 3 \rangle \langle 3, 1 \rangle} \end{pmatrix}$$

$$5 \text{ particles: } \begin{pmatrix} \frac{\langle 1, 2 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} & 0 & 0 & \frac{1}{\langle 3, 1 \rangle} \\ -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 1, 3 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} & 0 & 0 \\ 0 & -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 2, 4 \rangle}{\langle 2, 3 \rangle \langle 3, 4 \rangle} & \frac{1}{\langle 3, 4 \rangle} & 0 \\ 0 & 0 & -\frac{1}{\langle 3, 4 \rangle} & \frac{\langle 3, 5 \rangle}{\langle 3, 4 \rangle \langle 4, 5 \rangle} & \frac{1}{\langle 4, 5 \rangle} \\ -\frac{1}{\langle 4, 5 \rangle} & 0 & 0 & -\frac{1}{\langle 4, 5 \rangle} & \frac{\langle 4, 1 \rangle}{\langle 4, 5 \rangle \langle 5, 1 \rangle} \end{pmatrix}$$

$$3 \text{ particles: } \begin{pmatrix} \frac{\langle 1, 2 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} & \frac{1}{\langle 3, 1 \rangle} \\ -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 1, 3 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle} & \frac{1}{\langle 2, 3 \rangle} \\ -\frac{1}{\langle 3, 1 \rangle} & -\frac{1}{\langle 2, 3 \rangle} & \frac{\langle 2, 1 \rangle}{\langle 2, 3 \rangle \langle 3, 1 \rangle} \end{pmatrix}$$

Determinant

next page

$$\begin{aligned} \det Q_4 &= \begin{vmatrix} \frac{\langle 2, 4 \rangle}{\langle 2, 3 \rangle \langle 3, 4 \rangle} & \frac{1}{\langle 3, 4 \rangle} \\ -\frac{1}{\langle 3, 4 \rangle} & \frac{\langle 3, 1 \rangle}{\langle 3, 4 \rangle \langle 4, 1 \rangle} \end{vmatrix} = \frac{\langle 2, 4 \rangle \langle 3, 1 \rangle}{\langle 2, 3 \rangle \langle 3, 4 \rangle \langle 4, 1 \rangle} - \frac{1}{\langle 3, 4 \rangle^2} \\ &= \frac{1}{\langle 3, 4 \rangle^2} \left(\frac{\langle 2, 4 \rangle \langle 3, 1 \rangle}{\langle 2, 3 \rangle \langle 4, 1 \rangle} - 1 \right) \\ &= \frac{1}{\langle 3, 4 \rangle^2} \left(\frac{\langle 1, 2 \rangle \langle 3, 4 \rangle}{\langle 2, 3 \rangle \langle 4, 1 \rangle} - 1 \right) \\ &= -\frac{\langle 1, 2 \rangle}{\langle 3, 4 \rangle \langle 2, 3 \rangle \langle 4, 1 \rangle} = \frac{(-1)^{i+1} \langle 1, h-i \rangle}{\langle h-i, h-i+1 \rangle \dots \langle h, 1 \rangle} \\ &= \frac{\langle 1, 4-2 \rangle}{\langle 4-2, 4-2+1 \rangle \dots \langle 4, 1 \rangle} \\ &= -\frac{\langle 1, 2 \rangle}{\langle 2, 3 \rangle \langle 3, 4 \rangle \langle 4, 1 \rangle} \end{aligned}$$

$$\begin{aligned} \det Q_4 &= \frac{\langle 3, 2 \rangle}{\langle 3, 1 \rangle \langle 1, 2 \rangle} \left(\frac{\langle 4, 3 \rangle \langle 2, 1 \rangle}{\langle 3, 4 \rangle \langle 2, 3 \rangle \langle 1, 4 \rangle} - \frac{1}{\langle 2, 3 \rangle^2} \right) - \frac{1}{\langle 4, 2 \rangle} \left(\frac{1}{\langle 2, 3 \rangle \langle 3, 1 \rangle} + \frac{\langle 4, 1 \rangle}{\langle 1, 2 \rangle \langle 2, 3 \rangle \langle 3, 1 \rangle} \right) \\ &= \frac{\langle 3, 2 \rangle}{\langle 3, 1 \rangle \langle 1, 2 \rangle \langle 2, 3 \rangle^2} - \frac{\langle 3, 2 \rangle}{\langle 2, 3 \rangle^2 \langle 3, 1 \rangle \langle 1, 2 \rangle} - \frac{1}{\langle 4, 2 \rangle \langle 2, 3 \rangle \langle 3, 1 \rangle} - \frac{\langle 2, 1 \rangle}{\langle 1, 2 \rangle^2 \langle 2, 3 \rangle \langle 3, 1 \rangle} \\ &= -\frac{1}{\langle 3, 1 \rangle \langle 1, 2 \rangle \langle 2, 3 \rangle} - \frac{\langle 4, 3 \rangle}{\langle 3, 1 \rangle^2 \langle 4, 1 \rangle \langle 2, 3 \rangle} \\ &= -\frac{1}{\langle 3, 1 \rangle \langle 1, 2 \rangle \langle 2, 3 \rangle} + \frac{1}{\langle 2, 3 \rangle \langle 3, 1 \rangle \langle 1, 2 \rangle} - \frac{1}{\langle 1, 2 \rangle \langle 2, 3 \rangle \langle 3, 1 \rangle} + \frac{1}{\langle 1, 2 \rangle \langle 2, 3 \rangle \langle 3, 1 \rangle} \\ &= 0 \end{aligned}$$

Chapter 6

Ex 6.1

$$\text{Re}(l \cdot p_2) = (l \cdot p_1) = \frac{1}{2} [(l+p_1)^2 - l^2]$$

$$= \frac{1}{2} [l^2 + 2l \cdot p_1 + p_1^2 - l^2]$$

Mandelstam

$$\frac{(l+p_1)^2}{2l^2(l^2-p_1^2)(l^2-p_2^2)(l+p_1)^2} = \frac{1}{2l^2(l+p_1-p_2)^2(l-p_1)^2}$$

$\downarrow l \rightarrow -l$

$$2l^2(-l-p_1)^2(-l-p_2)^2(-l+p_1)^2$$

$$2l^2(l+p_1)^2(l+p_1+p_2)^2(l-p_1)^2$$

$$\text{so } l \cdot p_2 = \frac{1}{2} [(l+p_2)^2 - l^2]$$

$$(l-p_1-p_2)^2 = l^2 - 2l \cdot p_1 - 2l \cdot p_2 - 2p_1 \cdot p_2 + p_1^2 + p_2^2$$

$$(l-p_1)^2 = l^2 - 2l \cdot p_1 + p_1^2$$

$$(l-p_2)^2 = l^2 - 2l \cdot p_2 + p_2^2$$

$$(l-p_1)^2 + (l-p_2)^2 - l^2 - 2p_1 \cdot p_2 = (l-p_1-p_2)^2$$

$$(l+p_2)^2 = (l-p_1-p_2)^2$$

$$(l-p_2)^2 = l^2 - 2l \cdot p_2 + p_2^2$$

$$p_1 \rightarrow p_2 + k \quad (l-p_1-p_2)^2 \rightarrow (l-k)^2$$

$$(l-p_1)^2 \rightarrow (l+p_2-k)^2$$

$$p_2 = -p_1 - p_3$$

$$= -k - p_3$$

challenge: you really need an extra 5th factor of $(l-p_1)^2$ in the denominator - don't know how to get that

$$\text{WCS: } \frac{(l \cdot p_2)^2}{l^2(l-p_1)^2(l-p_2)^2(l-p_1-p_2)^2} \sim \frac{1}{k^2(k-k_1)^2(k-k_1-p_2)^2(k-k_1-p_2-k_3)^2}$$

$$l \rightarrow k - p_2$$

$$k^2$$

$$p_1 \rightarrow -p_2 + k$$

$$l-p_1 \rightarrow l+p_2-k$$

$$l-p_2 \rightarrow l-k$$

$$p_1 \rightarrow p_2 - k$$

$$l \rightarrow l - p_2 + k$$

$$(l-p_2+k)^2 = (l-p_2)^2 + k^2 - 2(l-p_2) \cdot k$$

my best shot

$$\frac{(l-p_2)^2}{l^2(l-p_1)^2(l-p_2)^2(l-p_1-p_2)^2} \xrightarrow{p_1 \rightarrow p_2+k} \frac{(l-p_2)^2}{l^2(l-p_2+k)^2(l-p_2)^2(l-p_2)^2} = \frac{(l-p_2)^2}{l^2(l-p_2+k)^2(l-p_2)^2(l-p_2)^2}$$

B.6.2 $C_4 I_4 + C_3 I_3 + C_2 I_2$

$$C_4 I_4 = A_4^{\text{tree}} \gamma_F \left[\frac{-u^2 s^2 (u^2 + s^2)}{t^4} \left(\frac{1}{su} \left(\frac{2}{\epsilon} (-u^2 s)^{-\epsilon} + (-u^2 u)^{-\epsilon} \right) - \ln^2 \left(\frac{s}{u} \right) - \pi^2 \right) \right. \\ \left. - \frac{su(s^2 + u^2)}{t^4} \left(\frac{2}{\epsilon} (-u^2 s)^{-\epsilon} + (-u^2 u)^{-\epsilon} \right) - \ln^2 \left(\frac{s}{u} \right) - \pi^2 \right] + \mathcal{O}(\epsilon)$$

$$A_4^{\text{tree}} \gamma_F \left((-1)^{1+\epsilon} \frac{2}{\epsilon} \frac{s^{1+\epsilon} u (s^2 + u^2) + su^{1+\epsilon} (s^2 + u^2)}{t^4} + \frac{su(s^2 + u^2)}{t^4} \left[\ln^2 \left(\frac{s}{u} \right) + \pi^2 + \mathcal{O}(\epsilon) \right] \right)$$

$su(s^{2+\epsilon} + s^\epsilon u^2 + s^2 u^\epsilon + u^{2+\epsilon})$
 $s^{3+\epsilon} u + s^{1+\epsilon} u^3 + s u^{3+\epsilon} + su^{3+\epsilon}$

$$C_3 I_3 = A_4^{\text{tree}} \frac{us(u^3 + us^2)}{t^4} \cdot \frac{\gamma_F}{\epsilon^2} (-u^2 s)^{-1-\epsilon} + \mathcal{O}(\epsilon)$$

$$= A_4^{\text{tree}} \frac{us^{-\epsilon}(u^3 + us^2)}{t^4} \frac{\gamma_F}{\epsilon^2} (-u^2)^{1+\epsilon} + \mathcal{O}(\epsilon)$$

$$\therefore \frac{u^4 s^{-\epsilon} + u^2 s^{2-\epsilon}}{t^4} = u^2 s^{-\epsilon} [u^2 + s^2]$$

$$C_2 I_2 = A_4^{\text{tree}} \frac{2u(s^2 + st + u + 2u^2)}{6t^3} \cdot \gamma_F \left(\frac{1}{\epsilon} - \ln(-u^2 s) + 2 \right) + \mathcal{O}(\epsilon)$$

$$= A_4^{\text{tree}} \gamma_F \frac{2u(5s^2 + st + 2u^2)}{6t^3} - \frac{2u(5s^2 + st + 2u^2)}{6t^3} \ln \left(\frac{s}{u^2} \right) + 2$$

Answer

$$C_4 I_4 + C_3 I_3 + C_2 I_2 = A_4^{\text{tree}} \gamma_F \left[(-1)^{1+\epsilon} \frac{2}{\epsilon} \frac{su(s^{2+\epsilon} + u^{2+\epsilon} + s^\epsilon u^2 + s^2 u^\epsilon)}{t^4} + \frac{su(s^2 + u^2)}{t^4} \left[\ln^2 \left(\frac{s}{u} \right) + \pi^2 \right] \right. \\ \left. + \frac{u^2 s^{-\epsilon}}{t^4} \frac{(-u^2)^{1+\epsilon}}{\epsilon^2} + \frac{2u(5s^2 + st + 2u^2)}{6t^3} \left(\frac{1}{\epsilon} - \ln \left(\frac{s}{u^2} \right) + 2 \right) \right] + \mathcal{O}(\epsilon)$$

Handwritten notes and diagrams below the answer box:

- Diagram showing a vertex with four external lines labeled s, t, u and a loop.
- Equations for propagator powers: s^{-1}, t^{-1}, u^{-1} and their combinations with ϵ .
- Equation for the loop integral: $\int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 (k+s)^2 (k+t)^2 (k+u)^2}$.
- Equation for the tree-level amplitude: $A_4^{\text{tree}} = \frac{1}{s^2 t^2 u^2}$.

BT p 130 (6.37) $\frac{\delta^{(8)}(\vec{Q})}{\langle 12 \rangle \langle 34 \rangle} \int d^4 n_1 d^4 n_2 \frac{\delta^{(8)}(P)}{\langle l_1 \rangle \langle 2l_2 \rangle \langle l_2 l_1 \rangle \langle l_3 \rangle \langle 4l_4 \rangle \langle l_4 l_3 \rangle}$

$\delta^{(8)}(P) = -1 \langle l_2 \rangle n_{12} + 1 \langle 3 \rangle n_3 + 1 \langle 4 \rangle n_4 + 1 \langle l_1 \rangle n_{l_1}$

RM BT p 18 p 86 $\delta^{(8)}(\vec{Q}) = \frac{1}{2^4} \prod_{i=1}^4 \sum_{j=1}^4 \langle ij \rangle n_{iA} n_{jA}$

$2 \langle ij \rangle n_{iA} n_{jA} = \langle 12 \rangle n_{1A} n_{2A} + \langle 21 \rangle n_{2A} n_{1A} - \langle 13 \rangle n_{1A} n_{3A} - \langle 31 \rangle n_{3A} n_{1A} + \langle 14 \rangle n_{1A} n_{4A} + \langle 41 \rangle n_{4A} n_{1A} - \langle 23 \rangle n_{2A} n_{3A} - \langle 32 \rangle n_{3A} n_{2A} + \langle 24 \rangle n_{2A} n_{4A} + \langle 42 \rangle n_{4A} n_{2A} - \langle 34 \rangle n_{3A} n_{4A} - \langle 43 \rangle n_{4A} n_{3A}$

↓ BLFW CRPP

$= \frac{1}{2^4} \prod_{A=1}^4 \left(2 \langle 12 \rangle \hat{n}_{1A} n_{2A} + 2 \sum_{j=3}^4 \langle 1j \rangle \hat{n}_{1A} n_{jA} + 2 \sum_{j=3}^4 \langle 2j \rangle n_{2A} n_{jA} + \sum_{j=3}^4 n_{1A} n_{jA} \right)$

$2 \langle 12 \rangle \hat{n}_{1A} n_{2A}$

$2 \sum_{j=3}^4 \langle 1j \rangle (n_{1A} + 2 n_{2A}) n_{jA} + 2 \sum_{j=3}^4 \langle 2+j \rangle n_{2A} n_{jA}$

$= 2 \langle 12 \rangle \hat{n}_{1A} n_{2A}$

$= 2 \langle 12 \rangle (n_{1A} + 2 n_{2A})$

$= 2 \sum_{j=3}^4 \langle 1j \rangle n_{1A} n_{jA} + 2 \sum_{j=3}^4 \langle 2+j \rangle n_{2A} n_{jA}$

does not change

$\delta^{(8)}(P) = \frac{1}{2^4} \prod_{i=1}^4 \left[-\langle l_2 3 \rangle n_{l_2} n_3 - \langle l_2 4 \rangle n_{l_2} n_4 - \langle l_2 l_1 \rangle n_{l_2} n_{l_1} + \dots - \langle l_1 l_2 \rangle n_{l_1} n_{l_2} \right]$

see (a.9.8) p 84

$\int d^4 n_1 d^4 n_2 = \frac{1}{2^4} \frac{d}{d n_{l_1}} \frac{d}{d n_{l_2}} \frac{d}{d n_{l_3}} \frac{d}{d n_{l_4}} \frac{2^4 \langle l_1 l_2 \rangle^4 n_{l_1} n_{l_2} n_{l_3} n_{l_4} - n_{l_3}^4}{2^4}$

$= \langle l_1 l_2 \rangle^4$

so $\frac{\delta^{(8)}(\vec{Q})}{\langle 12 \rangle \langle 34 \rangle} \int d^4 n_1 d^4 n_2 \frac{\delta^{(8)}(P)}{\langle l_1 \rangle \langle 2l_2 \rangle \langle l_2 l_1 \rangle \langle l_3 \rangle \langle 4l_4 \rangle \langle l_4 l_3 \rangle}$

$= \frac{\delta^{(8)}(\vec{Q})}{\langle 12 \rangle \langle 34 \rangle} \frac{\langle l_1 l_2 \rangle^2 (-1)}{\langle l_1 \rangle \langle 2l_2 \rangle \langle l_2 l_1 \rangle \langle l_3 \rangle \langle 4l_4 \rangle \langle l_4 l_3 \rangle}$

$= - \frac{\delta^{(8)}(\vec{Q}) \langle l_1 l_2 \rangle^2}{\langle 12 \rangle \langle 34 \rangle \langle l_1 \rangle \langle 2l_2 \rangle \langle l_2 l_3 \rangle \langle 4l_4 \rangle} = A_{4,4}^{\text{tree}} [1234] = A_4^{\text{MHV}} = \frac{\delta^{(8)}(\vec{Q})}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \text{ p 84}$

✓

[1234] = [4321]

continuation of exercise 6.3 p 130

$$A_q^{\text{rec}}(123\phi) = \frac{8^{1/2}}{4} \frac{1}{(l_1 l_2 l_3 \phi)} \frac{-5u}{(l_2^2 + p_2^2)(l_1 + p_1)^2} \Big|_{l_1^2 = p_1^2 = 0}$$

$$(l_2^2 + 2l_2 \cdot p_2 + p_2^2)(l_1^2 + 2l_1 \cdot p_1 + p_1^2)$$

$$2l_2 \cdot p_2 = \langle l_2 2 \rangle \langle l_2 2 \rangle \quad \langle l_1 \phi \rangle \langle l_1 \phi \rangle$$

$$s = (p_1 + p_2)^2 = \langle 12 \rangle \langle 12 \rangle$$

$$\text{or } s = p$$

$$s = (p_1 + p_2)^2 = (l_1 + l_2)^2 = \langle l_1 l_2 \rangle \langle l_1 l_2 \rangle$$

$$u = (p_1 + p_3)^2 = \langle 13 \rangle \langle 13 \rangle = \langle 2\phi \rangle \langle 2\phi \rangle$$

unt:

$$\frac{\langle l_1 l_2 \rangle^2 \langle 14 \rangle \langle 23 \rangle \langle l_1 \phi \rangle \langle l_2 2 \rangle}{\langle l_1 1 \rangle \langle l_2 3 \rangle} = -5u \Big|_{l_1^2 = l_2^2 = 0}?$$

hence:

$$-\langle l_1 l_2 \rangle \langle l_1 l_2 \rangle \langle 13 \rangle \langle 13 \rangle$$

$$\langle l_2 2 \rangle \langle l_2 2 \rangle \langle l_1 \phi \rangle \langle l_1 \phi \rangle$$

$$(l_1 + l_2)^2 = l_1^2 + 2l_1 l_2 + l_2^2$$

$$\frac{\langle l_1 l_2 \rangle \langle 13 \rangle \langle 13 \rangle}{\langle l_2 2 \rangle \langle l_1 \phi \rangle}$$

$$l_1 + l_2 = p_3 + p_4 = p_1 + p_2$$

$$\langle 12 \rangle \langle 12 \rangle = \langle 3\phi \rangle \langle 3\phi \rangle = \langle l_1 l_2 \rangle \langle l_1 l_2 \rangle$$

$$p_1 + p_2 + p_3 + p_4 = 0$$

$$\langle 12 \rangle + \langle 3\phi \rangle + \langle l_1 l_2 \rangle = 0$$

$$\langle 12 \rangle \langle l_2 3 \rangle + \langle l_1 \phi \rangle \langle l_1 \phi \rangle = 0$$

$$\langle l_1 l_2 \rangle \langle 1 l_1 \rangle = \langle 12 \rangle \langle l_2 l_2 \rangle$$

$$\frac{\langle l_1 l_2 \rangle}{\langle l_2 2 \rangle} = \frac{\langle 12 \rangle}{\langle l_1 \phi \rangle}$$

$$\langle l_2 \rangle \langle 3\phi \rangle + \langle l_1 \phi \rangle \langle l_1 \phi \rangle = 0$$

$$\langle l_2 3 \rangle \langle 3\phi \rangle + \langle l_2 l_1 \rangle \langle l_1 \phi \rangle = 0$$

$$\langle l_2 3 \rangle \langle 3\phi \rangle = -\langle l_2 l_1 \rangle \langle l_1 \phi \rangle$$

$$\boxed{\langle l_1 \phi \rangle = \frac{\langle l_2 3 \rangle \langle 3\phi \rangle}{\langle l_1 l_2 \rangle}}$$

$$\frac{-\langle l_1 l_2 \rangle^2}{\langle l_2 2 \rangle \langle l_1 \phi \rangle \langle l_1 \phi \rangle \langle l_2 3 \rangle \langle 3\phi \rangle}$$

$$\frac{\langle l_1 l_2 \rangle^2}{\langle l_2 2 \rangle \langle l_1 \phi \rangle \langle l_1 \phi \rangle \langle l_2 3 \rangle}$$

$$\frac{\langle l_1 l_2 \rangle^2}{\langle l_2 2 \rangle \langle l_1 \phi \rangle \langle l_1 \phi \rangle \langle l_2 3 \rangle}$$

$$\frac{\langle 12 \rangle \langle 13 \rangle \langle 13 \rangle}{\langle 3\phi \rangle}$$

$$\text{unt: } \langle 14 \rangle \langle 23 \rangle$$

$$\frac{\langle 12 \rangle}{\langle 3\phi \rangle} u = \langle 14 \rangle \langle 23 \rangle$$

$$\text{If } u = \langle 14 \rangle \langle 14 \rangle \text{ then } \frac{\langle 12 \rangle \langle 14 \rangle}{\langle 3\phi \rangle} = \langle 23 \rangle?$$

$$\langle 12 \rangle \langle 14 \rangle = \langle 23 \rangle \langle 3\phi \rangle$$

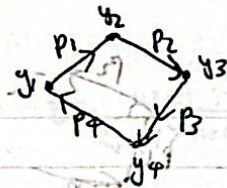
$$\langle 21 \rangle \langle 14 \rangle = \langle 23 \rangle \langle 3\phi \rangle$$

$$\langle 13 + 14 + 23 + 3\phi \rangle | \phi \rangle$$

✓ follows

BA p132 $s = -y_3^2 = -(y_1 - y_3)^2$, $u = -y_4^2 = -(y_2 - y_4)^2$

$y_1 - y_3 = p_1 - p_3$ $y_2 - y_4 = p_2 - p_4 = p_1 + p_4$



Ex 6.5
p136 $P_n^{MN}(E) = \exp \left[\sum_{L=1}^{\infty} \lambda^L (f^{(L)}(E) P_{q,1}^{MN}(L) + C^{(L)} + O(E)) \right]$
 $P_+^{MN}(E) = \exp \left[\sum_{L=1}^{\infty} \lambda^L (f^{(L)}(E) P_{q,1}^{MN}(L) + C^{(L)} + O(E)) \right]$

$L=1: \lambda (f^{(1)}(E) P_{q,1}^{MN}(1) + C^{(1)} + O(E))$
 $L=2: \lambda^2 (f^{(2)}(E) P_{q,1}^{MN}(2) + C^{(2)} + O(E))$
 $L=3: \lambda^3 (f^{(3)}(E) P_{q,1}^{MN}(3) + C^{(3)} + O(E))$
 $= \lambda f^{(1)}(E) P_{q,1}^{MN}(1) + \lambda C^{(1)} + \lambda^2 f^{(2)}(E) P_{q,1}^{MN}(2) + \lambda^2 C^{(2)} + \lambda^3 f^{(3)}(E) P_{q,1}^{MN}(3) + \lambda^3 C^{(3)}$
 $+ \frac{1}{2!} \left(\lambda^2 [f^{(1)}(E) P_{q,1}^{MN}(1)]^2 + 2 f^{(1)}(E) C^{(1)} P_{q,1}^{MN}(1) + C^{(1)2} \right) + 2 \lambda^3 (f^{(1)}(E) P_{q,1}^{MN}(1) + C^{(1)}) (f^{(2)}(E) P_{q,1}^{MN}(2) + C^{(2)})$
 $+ \frac{\lambda^3}{3!} [f^{(1)}(E) P_{q,1}^{MN}(1) + C^{(1)}]^3 + O(\lambda^4)$

$\lambda^2 \text{ terms: } \lambda^2 (f^{(2)}(E) P_{q,1}^{MN}(2) + C^{(2)} + \frac{1}{2} P_{q,1}^{MN}(1)^2 f^{(1)}(E)^2 + f^{(1)}(E) C^{(1)} P_{q,1}^{MN}(1) + \frac{1}{2} C^{(1)2})$
 $= \lambda^2 \left(\frac{1}{2} (P_{q,1}^{MN}(1))^2 + f^{(2)}(E) P_{q,1}^{MN}(2) + C^{(2)} \right)$

$\lambda^3 \text{ terms: } \lambda^3 (f^{(3)}(E) P_{q,1}^{MN}(3) + C^{(3)} + f^{(1)}(E) f^{(2)}(E) P_{q,1}^{MN}(1) P_{q,1}^{MN}(2) + C^{(1)} f^{(2)}(E) P_{q,1}^{MN}(2) + C^{(1)} f^{(1)}(E) P_{q,1}^{MN}(1) + \frac{1}{6} [f^{(1)}(E) P_{q,1}^{MN}(1) + C^{(1)}]^3)$
 $\frac{1}{6} P_{q,1}^{MN}(1)^3 = \frac{1}{2} P_{q,1}^{MN}(1)^3 - \frac{1}{3} P_{q,1}^{MN}(1)^3$

$= -\frac{1}{3} P_{q,1}^{MN}(1)^3 + \frac{1}{2} P_{q,1}^{MN}(1)^3 + P_{q,1}^{MN}(1) f^{(2)}(E) P_{q,1}^{MN}(2) + P_{q,1}^{MN}(1) C^{(2)} + f^{(3)}(E) P_{q,1}^{MN}(3) + C^{(3)}$
 $= -\frac{1}{3} P_{q,1}^{MN}(1)^3 + P_{q,1}^{MN}(1) \left[\frac{1}{2} P_{q,1}^{MN}(1)^2 + f^{(2)}(E) P_{q,1}^{MN}(2) + C^{(2)} \right] + f^{(3)}(E) P_{q,1}^{MN}(3) + C^{(3)} + O(E)$
 $= -\frac{1}{3} P_{q,1}^{MN}(1)^3 + P_{q,1}^{MN}(1) P_{q,2}^{MN}(E) + f^{(3)}(E) P_{q,1}^{MN}(3) + C^{(3)} + O(E)$

Chapter 7

p212 (10.40) δ [13456]

~~1345~~, 1456, 1346, 1356, 3456

δ [35612] 3561, 3562, 3512, 3612, 5612

δ [51234] = 5123, 1234, 5234, 5124, 5134

ans 0-2
 $0 = 5 \cdot 5 + 5 \cdot 5 + 5 \cdot 5 + 5 \cdot 5 + 5 \cdot 5 + 5 \cdot 5$
 $0 = \langle 0 | \text{PRES} \rangle + \langle 2 | \text{PRES} \rangle + \dots + \langle 2 | \text{PRES} \rangle$
 $0 = \langle 0 | \text{PRES} \rangle + \dots + \langle 2 | \text{PRES} \rangle$
 $0 = \langle 2 | \text{PRES} \rangle + \dots + \langle 2 | \text{PRES} \rangle$

Ex 7.1 $[u_1]^a \equiv \langle i | \hat{a} y_i^a = \langle i | \hat{a} y_{i+1}^a$ (5.44)

$$y_i^a = \frac{|i\rangle^a [u_{i-1}]^a - |i-1\rangle^a [u_i]^a}{\langle i-1, i \rangle}$$

$$y_{i+1}^a = \frac{|i+1\rangle^a [u_i]^a - |i\rangle^a [u_{i+1}]^a}{\langle i, i+1 \rangle}$$

↓ shift 1

$$= \frac{|i+1\rangle^a ([u_i]^a + w [u_{i+1}]^a) - (|i\rangle^a + w |i+1\rangle^a) [u_{i+1}]^a}{\langle i+1, i+1 \rangle}$$

$$= \frac{\langle i+1 | + w \langle i+1 |}{\langle i+1, i+1 \rangle} [u_i]^a + \frac{\langle i+1 | + w \langle i+1 |}{\langle i+1, i+1 \rangle} w [u_{i+1}]^a - \frac{\langle i | + w \langle i+1 |}{\langle i+1, i+1 \rangle} [u_{i+1}]^a - \frac{\langle i | + w \langle i+1 |}{\langle i+1, i+1 \rangle} w [u_{i+1}]^a}{\langle i+1, i+1 \rangle}$$

$$= \frac{|i+1\rangle^a [u_i]^a - |i\rangle^a [u_{i+1}]^a}{\langle i, i+1 \rangle} \leftarrow \text{unshifted!}$$

$$y_i = \frac{|i-1\rangle^a [u_{i-2}]^a - |i-2\rangle^a [u_{i-1}]^a}{\langle i-2, i-1 \rangle}$$

there are no 1's, so clearly empty is unshifted

$$\hat{y}_i = \frac{|i\rangle^a [u_{i-1}]^a - |i-1\rangle^a [u_i]^a}{\langle i-1, i \rangle}$$

$$= \frac{|i\rangle^a [u_{i-1}]^a + w |i+1\rangle^a [u_{i-1}]^a - |i-1\rangle^a [u_i]^a - w |i-1\rangle^a [u_{i+1}]^a}{\langle i-1, i \rangle + w \langle i-1, i+1 \rangle}$$

$$y_i = y_i + z(1-i)C_i$$

$$= \frac{i-i > C_{i+1} - (i-i) > C_i}{< i-i, i >} + \frac{w < i-i, i+1 >}{< i-i, i > + w < i-i, i+1 >} \cdot \frac{1-i-i > C_i}{(1-i-i) < i-i, i >}$$

$$= \frac{[i-i] < i-i, i > + [i-i] < i-i, i+1 > + [i-i] < i-i, i+1 >}{< i-i, i > + w < i-i, i+1 >}$$

$$\frac{i-i > C_{i+1} - (i-i) > C_i}{< i-i, i >} + \frac{w}{< i-i, i > + w < i-i, i+1 >} \cdot \frac{1}{< i-i, i > + w < i-i, i+1 >}$$

$$\cdot \frac{< i-i, i > + w < i-i, i+1 >}{< i-i, i > + w < i-i, i+1 >}$$

$$= \frac{< i-i, i > i-i > C_{i+1} - < i-i, i > i-i > C_i + w < i-i, i+1 > i-i > C_{i+1} - w < i-i, i+1 > i-i > C_i}{< i-i, i > (< i-i, i > + w < i-i, i+1 >)}$$

$$+ \frac{w < i-i, i > i-i > C_{i+1} + w < i-i, i > i-i > C_i + w < i-i, i+1 > i-i > C_{i+1} - w < i-i, i+1 > i-i > C_i}{< i-i, i > (< i-i, i > + w < i-i, i+1 >)}$$

Schauen

$$< i-i, i > i-i > + < i-i, i+1 > i-i > + < i-i, i+1 > i-i > = 0$$

$$\Rightarrow < i-i, i > i-i > + < i-i, i+1 > i-i > = - < i-i, i > i-i >$$

$$= < i-i, i > i-i >$$

$$\frac{< i-i, i > i-i > C_{i+1} - < i-i, i > i-i > C_i + w < i-i, i+1 > i-i > C_{i+1} - w < i-i, i+1 > i-i > C_i}{< i-i, i > (< i-i, i > + w < i-i, i+1 >)}$$

$$= \frac{i-i > C_{i+1} - (i-i) > C_i + w i-i > C_{i+1} - w i-i > C_i}{< i-i, i > + w < i-i, i+1 >}$$

Same!

pt 2 Eq. (7.10)

$$y_{ij} = p_0 + \dots + p_{j-1}$$

$\hat{p}_{i-1} = y_{i-1} - \hat{y}_i \leftarrow$ so we needed this of y_i as given in this language notation?

$$\begin{aligned} &= y_{i-1} - (y_i + z(i-1) \epsilon_i) \\ &= y_{i-1} - y_i - z(i-1) \epsilon_i \\ &= p_{i-1} - z(i-1) \epsilon_i \end{aligned}$$

$$= (i-1) (\epsilon_i - z \epsilon_i)$$

7.2 (grammar stuff:

$$\hat{\chi}_{1A} = \chi_{1A} + w \chi_{i+1,A}$$

$$= \langle 1 | \theta_{1A} \rangle + w \langle i+1 | \theta_{1A} \rangle$$

$$= \langle 1 | \theta_{1A} \rangle + w \langle i+1 | \theta_{1A} \rangle$$

$$= (\langle 1 | + w \langle i+1 |) | \theta_{1A} \rangle$$

$$= (\langle 1 | + w \langle i+1 |) \left(\sum_{r=1}^i |r\rangle \eta_{rA} \right)$$

$$= \sum_{r=1}^{i-1} \langle 1 | r \rangle \eta_{rA} + w \sum_{r=1}^i \langle i+1 | r \rangle \eta_{rA}$$

$$= \langle 1 | \theta_{1A} \rangle \eta_{1A} + w \langle i+1 | \theta_{1A} \rangle \eta_{iA}$$

$$\hat{\eta}_{i+1} = \eta_{i+1} + z \eta_i$$

$$| \theta_{1A} \rangle = \sum_{r=1}^{i-1} |r\rangle \eta_{rA}$$

$$+ \dots + |i-2\rangle \eta_{i-2,A} + |i-1\rangle \eta_{i-1,A} + |i\rangle \eta_{iA} + |i+1\rangle \eta_{i+1,A} + \dots$$

$$\downarrow \text{shift}$$

$$|i-1\rangle (\eta_{i-1,A} + z \eta_{iA})$$

$$= \dots + |i-2\rangle \eta_{i-2,A} + |i-1\rangle \eta_{i-1,A} + (|i\rangle - z |i-1\rangle) \eta_{iA} + |i+1\rangle \eta_{i+1,A} + \dots$$

$y_i = p_i - p_{i+1}$ was def of dual values y_i
 $\theta_i = ?$ can't just be $\langle i | \eta_{i,A} \rangle$, otherwise $\langle i | \theta_i \rangle = \langle i | \eta_{i,A} \rangle = 0$

$$z = \frac{w \langle i+1, i \rangle}{\langle i+1, i \rangle + w \langle i+1, i+1 \rangle} \rightarrow \langle i+1, i \rangle z + z w \langle i+1, i+1 \rangle = w \langle i+1, i \rangle$$

$$z \langle i+1, i \rangle = w \langle i+1, i \rangle - w z \langle i+1, i+1 \rangle$$

$$w = \frac{z \langle i+1, i \rangle}{\langle i+1, i \rangle - z \langle i+1, i+1 \rangle}$$

By analogy: we had $\langle i+1, \theta_i \rangle = \langle i, \theta_i \rangle$

$$y_{i+1} = p_i + \dots + p_{i+1} \Rightarrow p_{i+1} = y_{i+1} - y_i$$

Now:

$$\langle \theta_i | \eta_{i,A} \rangle = \langle i | \eta_{i,A} \rangle + \dots + \langle i-1 | \eta_{i,A} \rangle \Rightarrow \langle i | \eta_{i,A} \rangle = \langle \theta_i | \eta_{i,A} \rangle - \langle \theta_{i-1} | \eta_{i,A} \rangle$$

Now we use:

$$\chi_{i,A} = \langle i | \theta_i \rangle = \langle i | \theta_{i+1,A} \rangle, \chi_{i+1,A} = \langle i+1, \theta_{i+1,A} \rangle = \langle i+1, \theta_{i+2,A} \rangle$$

$$\chi_{i,A} = \chi_{i,A} + w \chi_{i+1,A} = \chi_{i,A} + \frac{z \langle i+1, i \rangle}{\langle i+1, i \rangle - z \langle i+1, i+1 \rangle} \chi_{i+1,A}$$

$$\chi_{i+1,A} | \eta_{i+1} \rightarrow \eta_{i+1} + z \eta_i$$

$$\langle i+1 | \eta_{i+1,A} \rangle = \langle \theta_{i+1,A} | \eta_{i+1,A} \rangle - \langle \theta_{i,A} | \eta_{i+1,A} \rangle = \langle \theta_{i+1,A} | \eta_{i+1,A} \rangle - \langle \theta_{i,A} | \eta_{i+1,A} \rangle$$

$$\langle i+1 | \eta_{i+1,A} \rangle = \langle i+1 | \eta_{i+1,A} \rangle + z \langle i+1 | \eta_{i,A} \rangle = \langle \hat{\theta}_{i+1,A} | \eta_{i+1,A} \rangle - \langle \hat{\theta}_{i,A} | \eta_{i+1,A} \rangle$$

$$\langle i | \theta_{i+1,A} \rangle + \frac{z \langle i+1, i \rangle}{\langle i+1, i \rangle - z \langle i+1, i+1 \rangle} \langle i+1, \theta_{i+1,A} \rangle = \langle i | \theta_{i+1,A} \rangle + \frac{z \langle i+1, i \rangle}{\langle i+1, i \rangle - z \langle i+1, i+1 \rangle} \langle i+1, \theta_{i+1,A} \rangle$$

$$\text{Now compare } \chi_{i,A} | \eta_{i+1} + z \eta_i = \langle i | \theta_{i+1,A} \rangle | \eta_{i+1} + z \eta_i = \langle i | \hat{\theta}_{i+1,A} \rangle$$

$$\langle i | \hat{\theta}_{i+1,A} \rangle = \langle i | \theta_{i+1,A} \rangle - \langle i+1 | \eta_{i+1,A} \rangle + z \langle i+1 | \eta_{i,A} \rangle$$

$$= \langle i | \theta_{i+1,A} \rangle - \langle i+1 | \eta_{i+1,A} \rangle + z \langle i+1 | \eta_{i,A} \rangle = \langle i | \theta_{i+1,A} \rangle - \langle i+1 | \eta_{i+1,A} \rangle + z \langle i+1 | \eta_{i,A} \rangle$$

$$\langle i+1 | \eta_{i+1,A} \rangle = \langle i+1 | \eta_{i+1,A} \rangle + z \langle i+1 | \eta_{i,A} \rangle = \langle \hat{\theta}_{i+1,A} | \eta_{i+1,A} \rangle - \langle \hat{\theta}_{i,A} | \eta_{i+1,A} \rangle$$

$$\langle i | \hat{\theta}_{i+1,A} \rangle = \langle i | \theta_{i+1,A} \rangle - \langle i+1 | \eta_{i+1,A} \rangle + z \langle i+1 | \eta_{i,A} \rangle = \langle i | \theta_{i+1,A} \rangle - \langle i+1 | \eta_{i+1,A} \rangle + z \langle i+1 | \eta_{i,A} \rangle$$

if $\langle i | \psi \rangle$ looks like $\langle i | \psi \rangle = \langle i | \psi \rangle$ and $\langle i | \psi \rangle = \langle i | \psi \rangle$

We know $\eta_{i+1} \rightarrow \eta_i + \epsilon \eta_i$ $\langle i | \psi \rangle = \langle i | \psi \rangle$ $P_i = y_i - y_{i+1}$ $\langle i | \psi \rangle = \langle i | \psi \rangle$
 leads to $\langle i | \psi \rangle = \sum_{j=1}^n \langle i | j \rangle \eta_j \rightarrow \sum_{j=1}^n \langle i | j \rangle \eta_j = \langle i | \psi \rangle$ $\langle i | \psi \rangle = \langle i | \psi \rangle$
 "by way" $\Rightarrow \langle i | \psi \rangle = \langle i | \psi \rangle$

What if $\theta_i = \theta_i - \theta_{i+1}$? $\langle i | \psi \rangle = \langle i | \psi \rangle$ $\langle i | \psi \rangle = \langle i | \psi \rangle$
 $= -\langle i+1 | \eta_{i+1}$ $\langle i | \psi \rangle = \langle i | \psi \rangle$

Then $\langle \theta_i \rangle = -\langle i+1 | \eta_{i+1}$ $\langle \theta_i \rangle = \langle \theta_i \rangle$ $\langle \theta_i \rangle = \langle \theta_i \rangle$
 $\langle \theta_{i+1} \rangle = -\langle i+2 | \eta_{i+2}$ $\langle \theta_{i+1} \rangle = \langle \theta_{i+1} \rangle$ $\langle \theta_{i+1} \rangle = \langle \theta_{i+1} \rangle$

$\chi_{iA} \rightarrow \chi_{iA} + w \chi_{i+1A}$ $\chi_{iA} = \chi_{iA}$ $\chi_{iA} = \chi_{iA}$
 $= \chi_i +$ $\chi_{iA} = \chi_{iA}$ $\chi_{iA} = \chi_{iA}$

$\chi_i = \langle i | \theta_i \rangle = \langle i | \theta_{i+1} \rangle$ $\chi_i = \langle i | \theta_i \rangle$ $\chi_i = \langle i | \theta_i \rangle$
 ↓ let's calculate this $\eta_{i+1} \rightarrow \eta_i + \epsilon \eta_i$ $\chi_i = \langle i | \theta_i \rangle$ $\chi_i = \langle i | \theta_i \rangle$
 then $\theta_i = \theta_i$ $\theta_i = \theta_i$ $\theta_i = \theta_i$
 $= \sum_{r=1}^n \langle i | r \rangle \eta_r$ $\theta_i = \theta_i$ $\theta_i = \theta_i$

$\langle i | \theta_i \rangle = \sum_{r=1}^n \langle i | r \rangle \eta_r$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$
 $= \langle i, 1 \rangle \eta_1 + \langle i, 2 \rangle \eta_2 + \dots + \langle i, i-2 \rangle \eta_{i-2} + \langle i, i-1 \rangle \eta_{i-1}$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$

shift $\rightarrow \langle i, 1 \rangle \eta_1 + \dots + \langle i, i-1 \rangle \eta_{i-1} + \epsilon \langle i, i \rangle \eta_i$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$
 $\sum_{r=1}^n \langle i, r \rangle \eta_r = \langle i | \theta_i \rangle$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$

$= \langle i-1, i \rangle \langle i | \theta_i \rangle + w \langle i-1, i+1 \rangle \langle i | \theta_i \rangle + w \langle i, i+1 \rangle \langle i, i-1 \rangle \eta_i$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$
 $\langle i-1, i \rangle + w \langle i-1, i+1 \rangle$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$ $\langle i | \theta_i \rangle = \langle i | \theta_i \rangle$

Let numerator such that it equals: $\langle i-1, i \rangle + w \langle i-1, i+1 \rangle$ $\langle i-1, i \rangle + w \langle i-1, i+1 \rangle$ $\langle i-1, i \rangle + w \langle i-1, i+1 \rangle$
 $(\langle i-1, i \rangle + w \langle i-1, i+1 \rangle) (\chi_i + w \chi_{i+1})$ $\langle i-1, i \rangle + w \langle i-1, i+1 \rangle$ $\langle i-1, i \rangle + w \langle i-1, i+1 \rangle$
 $(\langle i | \theta_i \rangle + w \langle i+1 | \theta_{i+1} \rangle)$ $\langle i | \theta_i \rangle + w \langle i+1 | \theta_{i+1} \rangle$ $\langle i | \theta_i \rangle + w \langle i+1 | \theta_{i+1} \rangle$

$= \langle i-1, i \rangle \langle i | \theta_i \rangle + w \langle i-1, i+1 \rangle \langle i | \theta_i \rangle + w \langle i-1, i \rangle \langle i+1 | \theta_{i+1} \rangle + w^2 \langle i-1, i+1 \rangle \langle i+1 | \theta_{i+1} \rangle$ $\langle i | \theta_i \rangle + w \langle i+1 | \theta_{i+1} \rangle$ $\langle i | \theta_i \rangle + w \langle i+1 | \theta_{i+1} \rangle$

want to get this, don't see how to get w^2 (?) w^2 (?) w^2 (?)

next page

$$\begin{aligned} & \omega \langle i-1, i-1 | \langle i, \theta_i \rangle + \omega \langle i, i+1 | \langle i, i+1 \rangle \eta_i \\ &= \omega (-\langle i, i-1 | \langle i+1, i+1 | \langle i-1 \rangle \theta_i) + \omega \langle i, i+1 | \langle i, i+1 \rangle \eta_i \\ &= -\omega \langle i, i-1 | \langle i+1 \theta_i \rangle - \omega \langle i+1, i \rangle \langle i-1 \theta_i \rangle + \omega \langle i, i+1 | \langle i, i+1 \rangle \eta_i \\ &= -\omega \langle i, i-1 \rangle \sum_{r=1}^L \langle i+1, r \rangle \eta_r - \omega \langle i+1, i \rangle \langle i-1, \theta_i \rangle + \omega \langle i, i+1 | \langle i, i+1 \rangle \eta_i \end{aligned}$$

$$= \cancel{w \langle i, i-1 \rangle \langle \theta_{i+1}, i+1 \rangle} + w \langle i, i-1 \rangle \langle \theta_{i+1}, i+1 \rangle - w \langle i+1, i \rangle \langle i+1, \theta \rangle$$

$$= -w \underbrace{\langle i, i+1 \rangle \langle i+1, i \rangle}_{\substack{\downarrow \\ \text{spatial}}} n_i$$

$$= w(\overset{\text{split}}{G_{i-1,i}} G_{i+1} + G_{i+1,i} G_i) (\theta_{i+1} - \theta_i)$$

$$= \omega \left(\underbrace{\langle 1, i \rangle \langle 1, \theta_{i+1} \rangle}_{\chi_{i+1}} + \underbrace{\langle 1, i \rangle \langle 1, \theta_i \rangle}_{\chi_i} - \underbrace{\langle 1, i \rangle \langle 1, \theta_i \rangle}_{\chi_{i+1}} - \underbrace{\langle 1, i \rangle \langle 1, \theta_{i+1} \rangle}_{\chi_i} \right)$$

$$= \omega(\langle 1-i, 1 \rangle (X_{q+1} - X_{q+1})) + \langle 1+i, i \rangle (X_1 - X_1)$$

$$= 0 \quad (?)$$

p143 $\langle \psi, \hat{i} \rangle \stackrel{?}{=} \langle \psi, i \rangle$

$\langle \hat{i} | (i) + w | i+1 \rangle$

$\langle i+1, i \rangle + w \langle i+1, i+1 \rangle$

$= \langle i+1, i \rangle + \frac{z \langle i+1, i \rangle}{\langle i+1, i \rangle - z \langle i+1, i+1 \rangle}$

$= \frac{\langle i+1, i \rangle \langle i+1, i+1 \rangle - z \langle i+1, i+1 \rangle \langle i+1, i \rangle + z \langle i+1, i \rangle \langle i+1, i+1 \rangle}{\langle i+1, i \rangle - z \langle i+1, i+1 \rangle}$

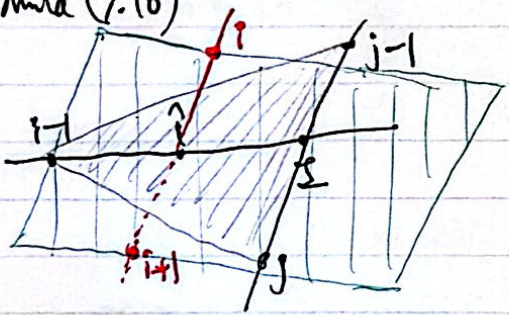
$\frac{1-i \langle i+1, i+1 \rangle + 1-i \langle i+1, i \rangle + 1-i \langle i+1, i+1 \rangle}{1-i \langle i+1, i+1 \rangle + 1-i \langle i+1, i \rangle + 1-i \langle i+1, i+1 \rangle}$

$= z \langle i+1, i \rangle (-1-i \langle i+1, i \rangle - 1-i \langle i+1, i+1 \rangle)$

$= -z \langle i+1, i \rangle \langle i+1, i \rangle - z \langle i+1, i+1 \rangle \langle i+1, i \rangle$

$= \frac{\langle i+1, i \rangle \langle i+1, i+1 \rangle}{\langle i+1, i \rangle - z \langle i+1, i+1 \rangle}$

Formula (7.16)



$(i+1, i) \langle i+1, i+1 \rangle + (i+1, i+1) \langle i+1, i \rangle = 0$

Formula (7.34)

$$\frac{w_x}{y_j^2} \frac{1}{w w_x} = \frac{\langle j, i-1 \rangle \langle j, i+1 \rangle}{\langle i, i-1 \rangle \langle j, j+1 \rangle} \cdot \left(-\frac{\langle i, i-1 \rangle \langle j, j+1 \rangle}{\langle i, i+1 \rangle \langle j, j-1 \rangle} \right) \cdot \frac{\langle i, i+1 \rangle \langle j, j-1 \rangle}{\langle i, i-1 \rangle \langle j, j+1 \rangle} = \frac{\langle i, i+1 \rangle \langle j, j-1 \rangle}{\langle i, i-1 \rangle \langle j, j+1 \rangle}$$

$$\frac{1}{w w_x} = \frac{1}{w + \frac{\langle i, i+1 \rangle \langle j, j-1 \rangle}{\langle i, i-1 \rangle \langle j, j+1 \rangle}} = \frac{\langle i, i-1 \rangle \langle j, j+1 \rangle}{\langle i, i+1 \rangle \langle j, j-1 \rangle w + \langle i, i-1 \rangle \langle j, j+1 \rangle}$$

$$0 = \frac{d}{dz} = -\frac{\langle i, i+1 \rangle \langle j, j-1 \rangle}{\langle i, i-1 \rangle \langle j, j+1 \rangle}$$

$$1 = \frac{d}{dz} = \frac{\langle i, i+1 \rangle \langle j, j-1 \rangle w + \langle i, i-1 \rangle \langle j, j+1 \rangle}{\langle i, i-1 \rangle \langle j, j+1 \rangle}$$

Formula (7.25)

$$(7.8) \frac{w \langle i, i+1 \rangle}{\langle i, i+1 \rangle + w \langle i, i+1 \rangle} = z \rightarrow w \langle i, i+1 \rangle = z (\langle i, i+1 \rangle + w \langle i, i+1 \rangle)$$

$$= z \langle i, i+1 \rangle + z w \langle i, i+1 \rangle$$

$$w \langle i, i+1 \rangle - z w \langle i, i+1 \rangle = z \langle i, i+1 \rangle$$

$$w (\langle i, i+1 \rangle - z \langle i, i+1 \rangle) = z \langle i, i+1 \rangle$$

$$w = \frac{z \langle i, i+1 \rangle}{\langle i, i+1 \rangle - z \langle i, i+1 \rangle} = \frac{z \langle i, i+1 \rangle}{\langle i, i+1 \rangle (1 - z)} = \frac{z}{1 - z}$$

$$dw = \frac{dz \langle i, i+1 \rangle}{\langle i, i+1 \rangle - z \langle i, i+1 \rangle} + \frac{z \langle i, i+1 \rangle}{(\langle i, i+1 \rangle - z \langle i, i+1 \rangle)^2} (-dz \langle i, i+1 \rangle)$$

$$= \frac{dz \langle i, i+1 \rangle (\langle i, i+1 \rangle - z \langle i, i+1 \rangle) + z dz \langle i, i+1 \rangle \langle i, i+1 \rangle}{(\langle i, i+1 \rangle - z \langle i, i+1 \rangle)^2}$$

$$= \frac{dz (\langle i, i+1 \rangle \langle i, i+1 \rangle - z \langle i, i+1 \rangle \langle i, i+1 \rangle + z \langle i, i+1 \rangle \langle i, i+1 \rangle)}{(\langle i, i+1 \rangle - z \langle i, i+1 \rangle)^2}$$

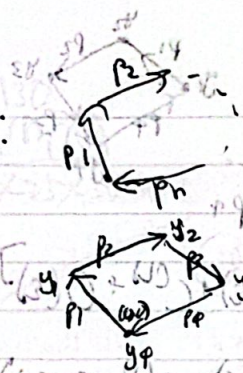
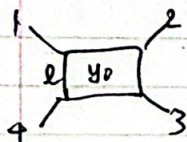
$$= \frac{dz \langle i, i+1 \rangle \langle i, i+1 \rangle}{(\langle i, i+1 \rangle - z \langle i, i+1 \rangle)^2}$$

$$\frac{dw}{w} = \frac{dz \langle i, i+1 \rangle \langle i, i+1 \rangle}{(z-1)^2 \langle i, i+1 \rangle^2} \cdot \left(\frac{\langle i, i+1 \rangle}{\langle i, i+1 \rangle} \cdot \frac{z-1}{z} \right)$$

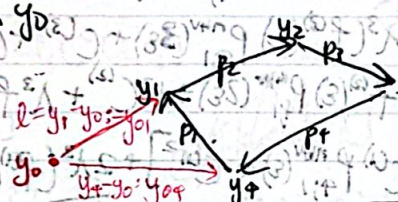
$$= \frac{\langle i, i+1 \rangle}{\langle i, i+1 \rangle} \cdot \frac{1}{z(z-1)} dz$$

$$= \frac{1}{z(z-1)} dz$$

(Dual) parameter space:



y_0 can be any; integrated on all of $\mathbb{R}^3$
Let's take a radon y_0



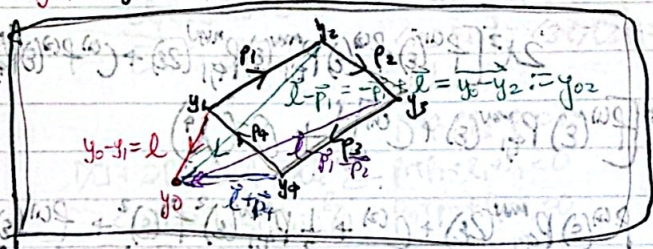
Idea: $L \Leftrightarrow y_0$

just this ~~is~~ p

$$p_1 \rightarrow y_1$$

$l \rightarrow y_0$
to a and

associates $L \rightarrow y_0$ a point in (dual) momentum space



dual space:
 point $\leftrightarrow$ surface
 line $\leftrightarrow$ dual line
 surface $\leftrightarrow$ point

BA p 142 7.2 wu:

$$\hat{\eta}_{i+1} = \eta_{i+1} + \varepsilon \eta_i \iff \chi_{i+1,A} = x_{i,A} + w \chi_{i-1,A}$$

28. $|\theta_A\rangle = |\theta_{1A}\rangle = |1\rangle \eta_{1A}$, $\chi_1^A = \langle \theta_{1A} \rangle = \langle \theta_{1A} | \theta_{1A} \rangle$, $\chi_{11}^A = \langle \eta_{1A} | \theta_{11A} \rangle = \langle 1 | \theta_{11A} \rangle$

$$L(|0_{i+1}\rangle - |0_i\rangle) = 1 - 7\eta_M$$

$$\langle i | \theta_{1A} \rangle - \langle i | \theta_{1A} \rangle = \frac{1}{0} \eta_{1A} \Rightarrow \langle i | \theta_{1A} \rangle = \langle i | \theta_{1A} \rangle$$

$$1 \rightarrow n_{i-1} \mapsto 1 \rightarrow \hat{n}_{i-1} = 1 \rightarrow (n_{i-1} + 2n_i)$$

$$= 1 - \frac{1}{2}(\eta_{i-1} + \eta_i)$$

$$= (1-17)\eta_1 + (1-17)\eta_2$$

$$\hat{\chi}_{11A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + w \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

1977

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$

$$|\hat{\alpha}_1\rangle - |\hat{\alpha}_2\rangle = (1-i)\lambda_{1+} + \frac{w\lambda_{1+}}{\delta - 1 - i + w\lambda_{1+}} (1-i)\lambda_{1+} + (1-i)\lambda_{1-} + \frac{w\lambda_{1-}}{\delta - 1 - i + w\lambda_{1-}} (1-i)\lambda_{1-} + (1-i)\lambda_{2+} + \frac{w\lambda_{2+}}{\delta - 1 - i + w\lambda_{2+}} (1-i)\lambda_{2+} + (1-i)\lambda_{2-} + \frac{w\lambda_{2-}}{\delta - 1 - i + w\lambda_{2-}} (1-i)\lambda_{2-}$$

$$\chi_{1,0} = \langle \hat{\theta}_1 | \hat{\theta}_1 \rangle = \langle \hat{\theta}_1 | \hat{\theta}_1 \rangle - \langle \hat{\theta}_1 | \hat{\theta}_1 \rangle \eta_1 - \langle \hat{\theta}_1 | \hat{\theta}_1 \rangle \eta_1$$

Quesad 9 be Antel
all well?

$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ -1 & 1 \end{pmatrix}$

Does $\hat{\theta}_{1-1}$ change? $X_{1-1} = \langle 1 | \theta_{1-1} \rangle = \langle 1 | \theta_1 \rangle$
 $X_{1-2} = \langle 1 | \theta_{1-2} \rangle = \langle 1 | \theta_{1-1} \rangle \in$ true dot charge, so $|\hat{\theta}_{1-1}\rangle \approx a|1,2\rangle + b|1,4\rangle$
 but the dot charge
 $X_1 = \langle 1 | \theta_1 \rangle = \langle 1 | \theta_{1-1} \rangle$

can also try: $\hat{\chi}_1 = \chi_1 + w \chi_{1+1}$ $\langle \hat{\theta}_1 \rangle = \langle \theta_1 \rangle + w \langle \hat{\theta}_{1+1} \rangle = 0$ $\text{avg } |\theta_1 - \hat{\theta}_1| = (1-w) \hat{n}_1$

$$\hat{\eta}_{i-1} = \eta_{i-1} + \varepsilon \eta_i$$

$$\frac{10.7 - 10.5}{1.17} + z \frac{10.7 - 10.4}{1.17}$$

$$|\hat{\theta}_{i-1}\rangle - |\hat{\theta}_i\rangle = |1\rangle\langle 1| n_{i-1} \mapsto |1\rangle\langle 1| (n_{i-1} + \epsilon n_i) = |\hat{\theta}_{i-1}\rangle - |\hat{\theta}_i\rangle$$

$$? \langle \hat{\sigma}_i \rangle - \langle \hat{\sigma}_i \rangle = \langle \hat{\sigma}_i \rangle \eta_{i-1} + z \langle \hat{\sigma}_i \rangle \eta_i \rightarrow \text{Suppose no sum} \quad \langle \hat{\sigma}_i \rangle - \hat{\chi}_i = \langle \hat{\sigma}_i \rangle \eta_{i-1} + z \langle \hat{\sigma}_i \rangle \eta_i \Rightarrow -\langle \hat{\sigma}_i \rangle$$

$$\angle 1 = \angle 1 - \angle 1 - \angle 1 - \angle 1$$

Lang-Quadrat $\chi_{1A} = \langle \hat{\theta}_i \rangle = \langle \hat{\theta}_{i+1} \rangle$

$$\text{calculus: } \langle \theta | \psi \rangle = \langle 1 | \psi \rangle \eta_{1,1} + 2 \langle 1 | \psi \rangle \eta_{1,2}$$

since $\langle 1 \rangle = \langle 1 - \frac{1}{2} \rangle$

$\langle 111 \rangle = \langle 111 \rangle + \emptyset$: no change, vacu.

$$\langle \uparrow \hat{\theta}_{i-1} \rangle = \langle \uparrow \theta_{i-1} \rangle$$

$$= (k_{11} - z_{11}) \theta_{11}$$

$$= \langle 1014 \rangle - 7 \langle 1014 \rangle$$

$$= -w \langle 1 | 4 \rangle \langle 1 | 0 | 4 \rangle$$

$$04, 17 \leq w \leq 14, 11$$

assum $\hat{\theta}_{i-1} = \theta_{i-1}$

~~$$\langle i, 0, i \rangle - \lambda_1 = \langle i, 0, i \rangle - \langle i, 0 \rangle - z \langle i, 0 \rangle + z \langle i, 0, i \rangle$$~~

2-1-21

$$X_i - \lambda_i + \delta$$

吾等之...

$$\boxed{w \leq 1 + 17G + 17n}$$

$$\langle 1, 17 + w \leq 1, 1 + 1 \rangle$$

$$= \frac{w\langle i-1, i \rangle \langle i+1, i \rangle \langle i+1, i-1 \rangle \langle i \rangle (|Q| - |Q_{i+1}|)}{\langle i-1, i \rangle + w\langle i-1, i+1 \rangle}$$

$$= -w_{ij} \frac{\langle i-1, j \rangle \langle i+1, j \rangle}{\langle i, j \rangle^2 + w_{ij}^2}$$

$$\langle \hat{\theta}_{i-1} \rangle - \langle \hat{\theta}_i \rangle = \epsilon_i (1 - \gamma) m_{i-1} + \epsilon_i (1 - \gamma) \eta_i$$

$$(\langle 1| + 2\langle -1|)(|0_{-1}\rangle - |0_1\rangle) = (\langle 1| + 2\langle -1|)(|0_{-1}\rangle - |0_1\rangle) + 2$$

$$(-) \langle 110 | \hat{H}_S | 110 \rangle = \langle 110 | \hat{H}_S | 110 \rangle - \langle 110 | \hat{H}_S | 110 \rangle = 0$$

973-727-6265

$$-\lambda_i = -\lambda_i + \frac{w}{\sqrt{1+\lambda_i^2}}$$

$$(-\hat{x}_i + x_i) \cdot (\langle y_1, i \rangle + \omega \langle y_1, i+1 \rangle) = \dots$$

$$(\langle 1, 1 \rangle + \omega \langle 1, 1 \rangle) (-\hat{\chi}_1) + \langle 1, 1 \rangle \chi_1 +$$

$$X_1 = \frac{(1+j)7 - 2w}{(1+j)5 - w}$$

$$|\theta_i\rangle - |\theta_{i+1}\rangle = |\gamma_i\rangle$$

$$|0\rangle \rightarrow |\hat{0}\rangle = |\uparrow\rangle \hat{n}_i = |\downarrow\rangle (\eta_{+1} + \eta_{-1})$$

$$\textcircled{1} \lambda_i = \lambda_1 + w \lambda_{i+1}$$

$$(2) \langle \hat{\theta}_i \rangle = \langle \hat{\theta}_i \rangle + w \langle \theta_{i+1} \rangle$$

$$= \langle \psi_0 \rangle + \frac{\langle \psi_1, i \rangle}{\langle \psi_1, \psi_1 \rangle} \langle \psi_1, i \rangle$$

$$\langle \hat{\theta}_1 \rangle - z \langle \psi | \hat{\theta}_1 \rangle = \langle \hat{\theta}_1 \rangle + \dots \langle \hat{\theta}_1 \rangle = \langle \hat{\theta}_1 \rangle$$

(3) $\rightarrow$

$$\langle \psi | \hat{H} | \psi \rangle - \langle \psi | \hat{H} | \psi \rangle = \langle \psi | \hat{H} | \psi \rangle - \langle \psi | \hat{H} | \psi \rangle$$

$$\langle i | r | j \rangle \hat{n}_{i-1} - \langle i | r | j \rangle \hat{n}_{i+1} = \langle i | r | j \rangle \hat{n}_{i-1} + \frac{\langle i | r | j \rangle}{\langle i | r | j \rangle \langle i+1 | r | i+1 \rangle} \langle i+1 | r | i+1 \rangle$$

$$\hat{n}_{i+1} = n_{i+1} + \frac{2G_{i+1} \Delta t}{G_{i+1} - 2G_{i+1} \Delta t} \frac{1}{G_{i+1} - 2G_{i+1} \Delta t} \langle G_{i+1} \Delta t \rangle$$

$$\frac{\langle 1101 \rangle}{\langle 11 \rangle} - \frac{\langle 1101 \rangle}{\langle 110 \rangle - \langle 011 \rangle} \neq X = \eta_1 = \frac{1}{\sqrt{2}} |01\rangle |01\rangle - |11\rangle$$

$\Delta(1+\gamma)_{1+} = \Delta(1+\gamma)_{1+} + \frac{1}{2} \Delta(1+\gamma)_{1+} + \frac{1}{2} \Delta(1+\gamma)_{1+}$
 - ? odd issue?

$$\begin{aligned} \langle \pi | \pi \rangle &= \langle \pi | \theta \rangle - \langle \pi | \theta \rangle \\ \langle \pi | \pi \rangle &= \frac{\langle \pi | \theta \rangle - X \pi}{\pi} \end{aligned}$$

[illegible]

Wes (5) : $\langle i+1 \rangle (\eta_i - \eta_{i-1} + 2\eta_i)$

Suppose sign (+) :

Suppose sign $\oplus$:

$$\frac{\partial \mathcal{L}}{\partial \theta_1} + \frac{\partial \mathcal{L}}{\partial \theta_2} + A = -\frac{\partial \mathcal{L}}{\partial \theta_1} + \frac{\partial \mathcal{L}}{\partial \theta_2} \quad \text{or sign flip here}$$

$$E\langle \psi | H | \psi \rangle = E\langle \psi | H | \psi \rangle + E\langle \psi | H | \psi \rangle + E\langle \psi | H | \psi \rangle = (-E\langle \psi | H | \psi \rangle + E\langle \psi | H | \psi \rangle) (-)$$

$$A = \cancel{2G+0A} + \cancel{2G+0A} - \cancel{2G+0A} - \frac{2G+1G+0A}{1A+2G+1A}$$

$$-ZG(-i, \hbar) \langle 1, \hbar \rangle - Z^2 \langle 1, \hbar \rangle \langle 1, \hbar \rangle - ZG(-i) \langle 1, \hbar \rangle$$

$$b_2 = \frac{1}{2} \langle \psi | \hat{H} | \psi \rangle = \frac{1}{2} \langle \psi | \hat{H} | \psi \rangle + \frac{1}{2} \langle \psi | \hat{H} | \psi \rangle$$

$$Z \langle M_{141} \rangle \langle 10_{II} \rangle$$

$$\frac{z^2 \langle 1, n \rangle X_i + z^2 \langle 1, n \rangle \langle 1, n \rangle}{z \langle 1, n \rangle - z \langle 1, n \rangle} = z \langle 1, n \rangle (X_i + z \langle 1, n \rangle)$$

$\angle 1 + \angle 2 = \angle 4 + \angle 3$

$$\frac{171X \text{ (171)} \times W-X \left(= \frac{W}{\text{दाता}} \right) (X \text{ का } 3 \text{ का भाग)} + 171X \text{ (171)} \times W-X \left(= \frac{W \times X}{\text{दाता}} + \frac{W \times 3}{\text{दाता}} \text{ का भाग} \right)}{2}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3-2 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 7 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \end{pmatrix}$$

7.3

$$\begin{aligned} \hat{P} &= |1\rangle - z|1-1\rangle \\ &= |1\rangle - \frac{\langle 1-1|1\rangle}{\langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle} |1-1\rangle \\ &= |1\rangle - \frac{\omega(-\langle 1-1|1\rangle + \langle 1-1|1-1\rangle)}{\langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle} |1-1\rangle \\ &= |1\rangle + \frac{\omega \langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle}{\langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle} |1-1\rangle \\ &= \frac{\langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle + \omega \langle 1-1|1\rangle + \omega^2 \langle 1-1|1-1\rangle}{\langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle} |1-1\rangle \end{aligned}$$

$$\frac{\langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle + \omega \langle 1-1|1\rangle + \omega^2 \langle 1-1|1-1\rangle}{\langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle} |1-1\rangle$$

$$\langle 1-1|1\rangle + \omega \langle 1-1|1-1\rangle + \omega \langle 1-1|1\rangle + \omega^2 \langle 1-1|1-1\rangle = 0$$

Is $\langle 1| = \langle 1| - z\langle 1-1|$?

$$|p\rangle^a = \epsilon^{ab} \langle p|_b$$

$$\epsilon_{ia} \langle p|^a = \epsilon_{ia} \epsilon^{ab} \langle p|_b \Rightarrow \delta_a^b \langle p|_b$$

$$\Rightarrow \epsilon_{ia} \langle p|^a = \langle p|_i$$

$$\epsilon_{ia} = -\epsilon^{ia} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} : \epsilon_{12} = 1, \epsilon_{21} = -1$$

$$\text{so } \langle p|_1 = \epsilon_{12} \langle p|^2 = -\langle p|^2$$

$$\langle p|_2 = \epsilon_{21} \langle p|^1 = \langle p|^1$$

$$|1\rangle^a = |1\rangle^a - z|1-1\rangle^a \epsilon_{ab} \text{ or } \epsilon_{ba}$$

$$|1\rangle_x^a = |1\rangle_x^a - z|1-1\rangle_x^a \epsilon_{ab}$$

$$\langle 1\rangle_y^a \langle 1\rangle_x^a = \langle 1\rangle_y^a \langle 1\rangle_x^a - z \langle 1\rangle_y^a \langle 1-1\rangle_x^a$$

$$\text{or } \langle 1\rangle_y^a \langle 1\rangle_x^a = \langle 1\rangle_y^a \langle 1\rangle_x^a - z \langle 1-1\rangle_y^a \langle 1-1\rangle_x^a$$

$$\left(\langle 1-1|_1 \rangle - \langle 1-1|_2 \rangle - \langle 1-1|_3 \rangle \right) \omega \langle 1-1|_4 \rangle$$

$$z \langle 1-1|_1 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

$$\omega \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle - \langle 1-1|_1 \rangle \langle 1-1|_2 \rangle$$

Ex 7.5 $k=1$:

$$A_n^{NMHV} = A_n^{MHV} \left\{ Y_{n-1}^{(1)}(\dots, z_{i-1}, z_{i+1}, z_{i+2}, \dots) + \sum_{j=i+3}^{i-2} [i-1, i, i+1, j-1, j] \cdot \hat{Y}_{n_L}^{(k_L)}(z_i, z_j, z_{i+1}, \dots, z_{i-1}) \cdot \hat{Y}_{n_R}^{(k_R)}(z_i, z_j, z_{i+1}, \dots, z_{j-1}) \right\}$$

$k_L + k_R + 1 = k = 1$

so $k_L = k_R = 0$

We know that $Y^{(0)} = Y^{MHV} = 1$

Now, $Y_{n-1}^{(0)}(\dots)$ is defined to be the prefactor of the N^k MHV supergraph with this A_{n-1}^{MHV}

That is: $A_{n-1}^{NMHV} = A_{n-1}^{MHV} \cdot Y_{n-1}^{(0)}(\dots)$

We know $Y_{n-1}^{(0)} = \sum_{j=2}^{n-4} \sum_{k=j+2}^{n-2} [n-1, j-1, j, k-1, k]$ by 7.22

so we are missing: $j = n-3, \sum_{k=j+2}^{n-2}$ and $\sum_{j=2}^{n-4} k = n-1$

$$\sum_{j=2}^{n-4} \sum_{k=j+2}^{n-2} [n-1, j-1, j, n-2, n-1]$$

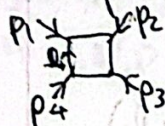
$\sum_{j=2}^{n-3} \sum_{k=j+2}^{n-1} [n, j-1, j, k-1, k]$

E.g. for $n=8$: $[1, 2, 3, 4], [1, 2, 4, 5], [1, 2, 5, 6], [2, 3, 4, 5], [2, 3, 5, 6], [3, 4, 5, 6]$

$n=6$: $[6, 1, 2, 3, 4], [6, 1, 3, 4, 5], [6, 2, 3, 4, 5], [7, 1, 2, 4, 5], [7, 1, 2, 5, 6], [7, 2, 4, 5, 6]$

$i=8$
 $j = i+3 \Rightarrow 1-2$
 $= 6 \Rightarrow 1$
 $[2, 3, 4, 5, 6], [2, 3, 4, 6, 7], [2, 3, 4, 7, 1]$

What is a "loop" in dual space?

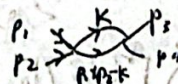


$$P_1 + P_2 + P_3 + P_4 = 0$$

$P_1 + P_2 + P_3 + P_4 = 0$
Two members not sum to zero? or sum the other is not

$$L_1 + (L_1 + p_1) + (L_1 + p_1 + p_2) + (L_1 + p_1 + p_2 + p_3) = 4L_1 + 3p_1 + 2p_2 + p_3$$

94-



Convex:

Constr: $l \uparrow$
 $p_4 \nearrow$ $l \rightarrow p_1 + p_2 + p_3$ we need: $p_4 + (l \rightarrow p_1 + p_2 + p_3) = l$

Ex 7.6 $\vec{z}_q = \vec{z}_1 + w \vec{z}_3$

$$\text{SD } \hat{\Sigma}_F \text{ known on } \mathbb{R}^n \text{ (i.e., } i, j+1) \Rightarrow |\hat{f}_i\rangle = |i\rangle + w|1\rangle$$

$$|\vec{\mu}_2\rangle = |\mu_1\rangle + \omega |\mu_1 - \rangle$$

$$\hat{X}_A = X_{1A} + w X_{1-1,A}$$

$$y_i^{aa} = \frac{\langle \hat{1} |^a \langle \hat{\mu}_M |^a - |1\rangle^a \langle \hat{\mu}_M |^a}{\langle 0 | \uparrow \rangle} = \frac{(1 + \omega |1\rangle) \langle \mu_M | - |1\rangle \langle \mu_M |}{\langle 1 - |1\rangle + \omega |1\rangle} \quad \text{+?}$$

$$= 1.7 [C_{\text{air}}] + \cancel{w \cdot 1.47 [C_{\text{air}}]} - 1.7 [C_{\text{air}}] - \cancel{w \cdot 1.47 [C_{\text{air}}]}$$

$\langle 14, i \rangle$

$$\hat{y}_{in} = \frac{\langle \hat{u}_i | \hat{u}_i \rangle}{\langle \hat{u}_i | \hat{u}_i \rangle} = \frac{1 + \langle \hat{u}_i | \hat{u}_i \rangle}{1 + \langle \hat{u}_i | \hat{u}_i \rangle} = 1$$

$$= \frac{|1+\rangle \langle 1+\rangle + \omega |1+\rangle \langle 1+\rangle + \omega |1+\rangle \langle 1+\rangle - \omega |1+\rangle \langle 1+\rangle}{\langle 1+\rangle + \omega \langle 1+\rangle + \omega \langle 1+\rangle} = \frac{|1+\rangle \langle 1+\rangle - \omega |1+\rangle \langle 1+\rangle}{\langle 1+\rangle + \omega \langle 1+\rangle + \omega \langle 1+\rangle} + \omega \frac{|1+\rangle \langle 1+\rangle - \omega |1+\rangle \langle 1+\rangle}{\langle 1+\rangle + \omega \langle 1+\rangle + \omega \langle 1+\rangle}$$

$$= y_{t+1} \cdot \frac{(y_{t+1})}{(y_{t+1})^2 + w_{t+1}^2} + w_{t+1} \frac{(w_{t+1})^2 - (y_{t+1})^2}{(y_{t+1})^2 + w_{t+1}^2}$$

$$\frac{1+\alpha \langle 1,1+\rangle [C_{11}] + \omega \hbar \langle 1,1+\rangle [C_{1-}] - i \langle 1,1+\rangle [C_{11}]}{\langle 1,1+\rangle (C_{11} + \omega \hbar C_{1-})} = \omega \sqrt{1+\alpha} \sqrt{1+\omega} [C_{11}]$$

not to use: $\frac{1}{2} \frac{d}{dt} \int_{\mathbb{R}^n} |u|^2 dx = \frac{1}{2} \frac{d}{dt} \int_{\mathbb{R}^n} |u|^2 dx + \frac{1}{2} \frac{d}{dt} \int_{\mathbb{R}^n} |u|^2 dx$

$$= |1+\rangle \langle 1+\rangle \langle 1+\rangle [u_1] + \omega |1+\rangle \langle 1+\rangle [u_2] - |1-\rangle \langle 1-\rangle [u_1] - \omega |1-\rangle \langle 1-\rangle [u_2]$$

So let add "Zero" = the neg for $+w(1+i)(i-1, n) | C_{n-1} - w(1+i)(i-1, n) | C_{n-1}$
 $(1+i) | C_{n-1} - w(1+i)(i-1, n) | C_{n-1} = (1+i) | C_{n-1} - w(1+i)(i-1, n) | C_{n-1}$

$$= \frac{|1+i\rangle\langle u_{n-1}| - |1\rangle\langle u_n|}{\sqrt{2}} + w|1+i\rangle \cdot (-i|1,1+i\rangle\langle u_n| + |1,1+i\rangle\langle u_{n-1}| + |1,1\rangle\langle u_{n-1}|)$$

$$= y_{11} + \frac{w_{11} + w_{12}}{2}$$

$$= y|1\rangle + z|1\rangle|0\rangle \quad \text{where } z = \frac{w}{\langle 1,1\rangle(\langle 1,1\rangle + w\langle 1,1\rangle)}$$

Thus, $\beta_t = y_t - \hat{y}_{t+1} = y_t - y_{t+1} - z_{t+1}^T \alpha = p_t - z_{t+1}^T \alpha = \eta^T \alpha - z_{t+1}^T \alpha = (\eta - z_{t+1})^T \alpha$

$$\hat{p}_{t+1} = \hat{y}_{t+1} - y_{t+2} = \hat{y}_{t+1} - y_{t+2} = y_{t+1} + \epsilon_{t+1} - y_{t+2}$$

$$|1\rangle \rightarrow |1'\rangle = |1\rangle - 8|1+1\rangle$$

$$\{E+1, 1\} \cup \{E+1\} = \{E+1\} \cup \{E\} \quad \{E+1, 1\} \cup \{E\}$$

by 7.65

Bp 7.7 $\langle AB\hat{4}1 \rangle \neq \langle AB41 \rangle - \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \langle AB\hat{3}1 \rangle$ no standard - 2 to work going up and down

$[2, 1, 1] + [1, 1, 1] = [3, 2, 2]$

$\langle AB12 \rangle \hat{4} + \langle B12\hat{4} \rangle A + \langle 12\hat{4}AB \rangle + \langle 2\hat{4}AB1 \rangle + \langle 4AB\hat{1} \rangle 2 = 0$

$\begin{cases} \hat{z}_4 = \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \hat{z}_3 \\ \hat{z}_B = -\frac{\langle B341 \rangle}{\langle A341 \rangle} \hat{z}_A + \hat{z}_B \end{cases}$

If the 3 terms vanish, then so does $\langle B12\hat{4} \rangle A$

$\langle AB12 \rangle \left(\hat{z}_4 - \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \hat{z}_3 \right) + \langle 124A \rangle \left(\hat{z}_B - \frac{\langle B341 \rangle}{\langle A341 \rangle} \hat{z}_A \right) - \langle 123A \rangle \cdot \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \left(\hat{z}_B - \frac{\langle B341 \rangle}{\langle A341 \rangle} \hat{z}_A \right)$

$+ \langle 24AB \rangle$

$\hat{4} \langle 24AB \rangle - \langle 23AB \rangle \cdot \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \cdot 1$

$= \left(\langle 24AB \rangle - \frac{\langle 23AB \rangle \cdot \langle 4AB1 \rangle}{\langle 3AB1 \rangle} \right) \cdot 1 - \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \cdot \langle AB12 \rangle \cdot 3 + \langle AB12 \rangle \cdot 4$

$+ \left(\frac{\langle B341 \rangle}{\langle A341 \rangle} \cdot \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \cdot \langle 123A \rangle - \frac{\langle B341 \rangle}{\langle A341 \rangle} \langle 124A \rangle \right) A + \left(\langle 124A \rangle - \langle 123A \rangle \cdot \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \right) B$

$\frac{\langle B341 \rangle}{\langle A341 \rangle} \left(\frac{\langle 4AB1 \rangle \langle 123A \rangle}{\langle 3AB1 \rangle} - \langle 124A \rangle \right) A$

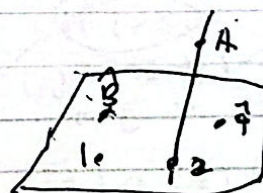
$\left(\frac{\langle 2341 \rangle}{\langle 2341 \rangle} + \frac{\langle 4AB1 \rangle}{\langle 3AB1 \rangle} \right) =$

Take another look:

$\langle A\hat{B}12 \rangle \hat{4} + \langle 12\hat{4}AB \rangle \hat{B} + \langle 2\hat{4}AB1 \rangle$

$= \langle A2B1 \rangle \hat{4} + \langle A21\hat{4} \rangle \hat{B} + \langle A2\hat{4}B \rangle = (A2) \cap (B1\hat{4})$

That is zero if $(A2)$ already lies in the plane $(B1\hat{4})$ (?)



Chapter 8

Equation (8.3) p 163

Let's do the $D=3$ case:

$$l^2 = (l-x)^2 - (l-y)^2 = 0 \quad \text{has 2 independent solutions?}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} l_0 \\ l_1 \\ l_2 \end{pmatrix}^2 = 0 \rightarrow \left(\begin{pmatrix} l_0 \\ l_1 \\ l_2 \end{pmatrix} - \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} \right)^2 = 0$$

$$l_0^2 - l_1^2 - l_2^2 = 0$$

$$(l_0 - x_0)^2 - (l_1 - x_1)^2 - (l_2 - x_2)^2 = 0$$

$$l_0^2 = l_1^2 + l_2^2$$

$$(l_0 - y_0)^2 - (l_1 - y_1)^2 - (l_2 - y_2)^2 = 0$$

Let's give the values:

$$x_0=1, x_1=0, x_2=2$$

$$y_0=2, y_1=1, y_2=0$$

$$(l_1^2 + l_2^2 - x_0)^2 - (l_1 - x_1)^2 - (l_2 - x_2)^2 = 0$$

$$(l_1^2 + l_2^2 - 1)^2 - l_1^2 - (l_2 - 2)^2 = 0$$

$$(l_1^2 + l_2^2)^2 - 2(l_1^2 + l_2^2) + 1 - l_1^2 - (l_2^2 - 4l_2 + 4) = 0$$

$$l_1^4 + 2l_1^2 l_2^2 + l_2^4 - 2l_1^2 - 2l_2^2 + 1 - l_1^2 - l_2^2 + 4l_2 - 4 = 0$$

$$l_1^4 + 2l_1^2 l_2^2 + l_2^4 - 3l_1^2 - 3l_2^2 - 3 = 0$$

$$(l_0 - y_0)^2 - (l_1 - y_1)^2 - (l_2 - y_2)^2 = 0$$

$$(l_0 - 2)^2 - (l_1 - 1)^2 - l_2^2 = 0$$

$$l_1^4 + 2l_1^2 l_2^2 + l_2^4 - 2l_1^2$$

$$l_0^2 - 2l_0 + 1 - l_1^2 - l_2^2 + 4l_2 - 4 = 0$$

$$l_0^2 - 4l_0 + 4 - l_1^2 - 2l_1 - 1 - l_2^2 = 0$$

$$l_0^2 = l_1^2 + l_2^2$$

$$\oplus l_0 = \pm \sqrt{l_1^2 + l_2^2}$$

2 solutions

$$l_1^2 + l_2^2 - 2\sqrt{l_1^2 + l_2^2} - 3 - l_1^2 - l_2^2 + 4l_2 = 0$$

$$l_1^2 + l_2^2 - 4\sqrt{l_1^2 + l_2^2} + 3 - l_1^2 - 2l_1 - l_2^2 = 0$$

don't see why this must be unique (given the 2 solutions)

Let's do the simplest case: 2D:

$$l^2 = (l - p_1)^2 = 0, \quad p_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$l_0^2 - l_1^2 = 0, \text{ let's call it } t^2$$

$$t^2 - x^2 = 0 \quad \& \quad \left(\begin{pmatrix} t \\ x \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)^2 = 0$$

$$t^2 - (x-1)^2 = 0$$

$$t^2 - x^2 = 0$$

$$t^2 - (x^2 - 2x + 1) = 0$$

$$t^2 - x^2 + 2x - 1 = 0$$

$$2x - 1 = 0$$

$$t^2 - \frac{1}{2} = 0$$

$$t^2 = \frac{1}{2}$$

$$t = \pm \frac{1}{\sqrt{2}} \quad x = \frac{1}{2}$$

Now let's do 3D with $p_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad p_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\text{so: } t^2 - x^2 - y^2 = 0, \quad t^2 - (x-1)^2 - y^2 = 0, \quad t^2 - x^2 - (y-1)^2 = 0$$

$$t^2 - x^2 + 2x - 1 - y^2 = 0$$

$$t^2 - x^2 - y^2 + 2y - 1 = 0$$

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

$$t^2 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 0$$

$$t^2 - \frac{1}{4} - \frac{1}{4} = 0$$

$$t^2 - \frac{1}{2} = 0$$

$$t^2 = \frac{1}{2}$$

$$t = \pm \frac{1}{\sqrt{2}} \quad (\text{2 solutions!})$$

$$x = \frac{1}{2}$$

$$y = \frac{1}{2}$$

Ex. 8.1 $LS_{(a)} = \int \prod_{i=1}^4 d^3 \eta_{l_i} (A_3^{\overline{mu}}(-l_1, p_1, l_1) A_3^{mu}(-l_1, p_2, l_2) A_3^{\overline{mu}'}(-l_2, p_3, l_3) A_3^{mu'}(-l_3, p_4, l_4))$

$$\frac{1}{[l_1, p_1][p_1, l_1][l_1, -l_1]} \prod_{A=1}^4 ([-l_1, p_1] \eta_{l_1 A} + [p_1, l_1] \eta_{-l_1 A} + [l_1, -l_1] \eta_{p_1 A}) \frac{1}{\langle -l_1, p_2 \rangle \langle p_2, l_2 \rangle \langle l_2, -l_2 \rangle} \prod_{B=1}^4 \langle -l_1, p_2 \rangle \eta_{l_2 B} \eta_{p_2 B} + \langle -l_2, l_2 \rangle \eta_{l_2 B} \eta_{l_2 B}$$

$$+ \langle p_3, l_2 \rangle \eta_{p_3 B} \eta_{l_2 B}$$

$$\frac{\partial}{\partial n_{l_4}} \frac{\partial}{\partial n_{l_3}} \frac{\partial}{\partial n_{l_2}} \frac{\partial}{\partial n_{l_1}} \prod (n_{l_4} + n_{p_1} + n_{l_1}) \prod (n_{l_3} + n_{p_2} + n_{l_2}) \prod (n_{l_2} + n_{p_3} + n_{l_3}) \prod (n_{l_1} + n_{p_4} + n_{l_4})$$

$A=1$: thing puts us $[-l_2, p_1]$, which now we need to get the $\langle -l_3, p_2 \rangle$ from the last elem, which now we get $[-l_2, p_3]$ and $\langle -l_1, p_2 \rangle$

$$\Rightarrow \frac{[-l_4, p_1] \langle -l_3, p_4 \rangle [-l_2, p_3] \langle -l_1, p_2 \rangle + [p_1, l_1] \langle p_2, l_2 \rangle [p_3, l_3] \langle p_4, l_4 \rangle}{[-l_4, p_1] [p_1, l_1] \langle -l_1, p_2 \rangle \langle p_2, l_2 \rangle [l_2, -l_1] [-l_2, p_3] [p_3, l_3] \langle l_3, -l_2 \rangle \langle -l_3, p_4 \rangle \langle p_4, l_4 \rangle \langle l_4, -l_3 \rangle} \neq 0 \quad \text{for } A=1,2,3,4$$

Redo: If we hit $N_{\ell+1}$, we prob up a $[p, l_1]$, so we need to hit N_{ℓ_1} from the MUV, then will prob up a $(\langle l_1, p_2 \rangle N_{p_2} + \langle l_1, l_2 \rangle N_{\ell_2})$. We need to hit N_{ℓ_2} , so we get it from the MUV, giving us $[p_3, l_3]$. Finally, we need to hit N_{ℓ_3} . $\langle l_3, p_4 \rangle N_{p_4} + \langle l_3, l_4 \rangle N_{\ell_4}$, then a tie goes to:

$$[p_1, l_1] \left(\langle l_1, p_4 \rangle \eta_{p_4,1} + \langle l_1, -l_3 \rangle \eta_{l_3,1} \right) [l_2, p_3] \left(\langle l_2, l_1 \rangle \eta_{l_1,1} + \langle l_2, p_2 \rangle \eta_{p_2,1} \right)$$

$$\frac{4}{A^2} \left([p_1, l_1] [l_3, p_3] \left(\langle l_1, p_1 \rangle \eta_{p_1, A} + \langle l_1, l_3 \rangle \eta_{l_3, A} \right) \langle l_3, l_1 \rangle \eta_{l_1, A} + \langle l_2, p_2 \rangle \eta_{p_2, A} \right) + [p_1, l_1] [p_3, l_3] \left(\langle l_1, p_2 \rangle \eta_{p_2, A} + \langle l_1, l_2 \rangle \eta_{l_2, A} \right) \langle l_2, l_1 \rangle \eta_{l_1, A} \langle l_3, p_3 \rangle$$

[illegible]

$$(10 \leq 110) \text{ cm} = 10 \text{ cm} \leq 110 \text{ cm} =$$

$$\begin{aligned} \langle 111|S-\langle S\rangle &= \langle S\rangle \leftarrow \langle S\rangle \\ |111\rangle \langle 111| &= |111\rangle \leftarrow |111\rangle \end{aligned}$$

Ex 8.2 $\langle A, B, 1, 1 \rangle = \det \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$

eq. $\langle A, B, 1, 2 \rangle = \det \begin{pmatrix} a_1 & b_1 + a_1 & b_2 + a_2 \\ a_2 & b_2 + a_2 & b_3 + a_3 \\ a_3 & b_3 + a_3 & b_4 + a_4 \end{pmatrix} = \det \begin{pmatrix} a_1 & b_1 & b_2 \\ a_2 & b_2 & b_3 \\ a_3 & b_3 & b_4 \end{pmatrix} = \det \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} = \det \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} \langle 1, 2, 3, 4 \rangle$

GL(2) $\Rightarrow \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} = 0$

$\begin{pmatrix} x & y \\ z & w \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ for given a_1, a_2 , this means $\begin{pmatrix} x & y \\ z & w \end{pmatrix}$ is degenerate, proof:

$xa_1 + ya_2 = 0 \Rightarrow xa_1 = -ya_2$
 $za_1 + wa_2 = 0 \Rightarrow za_1 = -wa_2$

$x = -\frac{b_2}{a_2} y$

$z = -\frac{b_2}{a_2} w$

$\Rightarrow \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} -\frac{b_2}{a_2} y & y \\ -\frac{b_2}{a_2} w & w \end{pmatrix}$ rows are all by a factor of $-\frac{w}{y}$, so are linearly dependent

so columns $\begin{pmatrix} x & y \\ z & w \end{pmatrix} \in GL(2)$

For GL(2) $\Rightarrow \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} x & y \\ z & w \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} xa_1 + ya_2 \\ za_1 + wa_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} xa_1 + ya_2 \\ za_1 + wa_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -\frac{b_2}{a_2} y & y \\ -\frac{b_2}{a_2} w & w \end{pmatrix}$

$\begin{pmatrix} -\frac{b_2}{a_2} w & w \end{pmatrix} a_1 + w b_2 = 1$

$w(b_2 - \frac{b_2}{a_2} a_1) = 1$

$w \left(\frac{b_2 a_2 - b_2 a_1}{a_2} \right) = 1 \Rightarrow w = \frac{a_2}{a_2 b_2 - a_1 b_2}$

$\begin{pmatrix} -\frac{b_2}{a_2} y & y \\ -\frac{b_2}{a_2} w & w \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Ex 8.3 $(\mathbf{p}_1)^2 = (-10\mathbf{i} + \frac{23}{13}\mathbf{j} + 17\mathbf{k})^2 = 100\mathbf{i}^2 + \frac{529}{169}\mathbf{j}^2 + 289\mathbf{k}^2 = 0$

$(\mathbf{p}_2)^2 = (-10\mathbf{i} + \frac{23}{13}\mathbf{j} + 17\mathbf{k})^2 = 100\mathbf{i}^2 + \frac{529}{169}\mathbf{j}^2 + 289\mathbf{k}^2 = 0$

$(\mathbf{p}_1 - \mathbf{p}_2)^2 = (-\frac{23}{13}\mathbf{j} + 17\mathbf{k})^2 = \frac{529}{169}\mathbf{j}^2 + 289\mathbf{k}^2 = 0$

$(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)^2 = (-\frac{23}{13}\mathbf{j} + 17\mathbf{k} + 13\mathbf{i})^2 = \frac{529}{169}\mathbf{j}^2 + 289\mathbf{k}^2 + 169\mathbf{i}^2 + 2 \cdot \frac{23}{13} \cdot 17 \cdot 13 = 0$

$-2 \cdot \frac{23}{13} \cdot 17 \cdot 13 + 2 \cdot 17 \cdot 13 = 2(169 - 169) = 0$

Similarly for $l^{(2)}$ the default are with last one

$$\begin{aligned} (l_1 - p_1 - p_2 - p_3)^2 &= -1 \langle 1 | \langle 1 | - \frac{[23]}{[13]} \langle 2 | \langle 1 | + 1 \langle 1 | \langle 1 | + 1 \langle 2 | \langle 2 | + 1 \langle 3 | \langle 3 | \\ &= \left(-\frac{[23]}{[13]} \langle 2 | \langle 1 | + 1 \langle 2 | \langle 2 | + 1 \langle 3 | \langle 3 | \right) = \frac{[23]^2}{[13]^2} \langle 2 | \langle 1 | + \langle 2 | \langle 2 | + \langle 3 | \langle 3 | - 2 \frac{[23]}{[13]} \langle 2 | \langle 2 | \\ &= \frac{[23]}{[13]} \langle 2 | \langle 3 | \langle 3 | + 2 \langle 2 | \langle 2 | \langle 3 | \\ &= 2 \langle 2 | \langle 3 | \langle 2 | - \langle 2 | \langle 2 | \langle 3 | \end{aligned}$$

little group scaling: $l^{(1)}: t, \lambda, t, \tilde{\lambda} \rightarrow \begin{pmatrix} 1 & 0 \\ t & -2hi \end{pmatrix}$

p_1 has zero little group weight
 p_2 has little group weight zero
 p_3 has little group weight zero

is both } this is what we want: an amplitude has little group weight
but a momentum has no little group weight Amor card for this?

$$l^{(1)} = -1 \langle 1 | \langle 1 | - 1 \langle 2 | \langle 2 | \frac{[23]}{[13]} \quad \begin{pmatrix} A & B \\ 1 & 3 \end{pmatrix} \Leftrightarrow \text{point in dual space}$$

$a\tilde{z} + b\tilde{z}_3$

$$l^{(1)} = -1 \langle 1 | \langle 1 | + \langle 2 | \langle 2 | \frac{[23]}{[13]} = \lambda_1 (\tilde{\lambda}_1 + \tilde{\lambda}_2 \frac{[23]}{[13]})$$

$$= \frac{\langle 2 | \langle 1 | \langle 1 | + \langle 1 | \langle 2 | \langle 2 | + \langle 3 | \langle 2 | \langle 2 | + \langle 2 | \langle 3 | \langle 3 | + \langle 1 | \langle 3 | \langle 3 |}{\langle 3 | \langle 1 | \langle 2 |} = \frac{\langle 2 | \langle 1 | \langle 1 | + \langle 1 | \langle 2 | \langle 2 | + \langle 3 | \langle 2 | \langle 2 | + \langle 2 | \langle 3 | \langle 3 | + \langle 1 | \langle 3 | \langle 3 |}{\langle 3 | \langle 1 | \langle 2 |}$$

$$= \frac{\langle 2 | \langle 1 | \langle 1 | + \langle 1 | \langle 2 | \langle 2 | + \langle 3 | \langle 2 | \langle 2 | + \langle 2 | \langle 3 | \langle 3 | + \langle 1 | \langle 3 | \langle 3 |}{\langle 3 | \langle 1 | \langle 2 |}$$

$$= \frac{\langle 2 | \langle 1 | \langle 1 | + \langle 2 | \langle 3 | \langle 3 | + \langle 3 | \langle 2 | \langle 2 | + \langle 1 | \langle 3 | \langle 3 |}{\langle 3 | \langle 1 | \langle 2 |}$$

$$\langle 3 | \langle 2 | \langle 2 | + \langle 2 | \langle 3 | \langle 3 | + \langle 1 | \langle 3 | \langle 3 | = 0$$

$$\langle 2 | \langle 1 | \langle 1 | + \langle 2 | \langle 3 | \langle 3 | + \langle 3 | \langle 2 | \langle 2 |$$

$$= \frac{\langle 1 | \langle 1 | + \langle 3 | \langle 3 | + \langle 3 | \langle 1 |}{\langle 3 | \langle 1 | \langle 2 |} \rightarrow \text{this is the } [1] \text{ for } l^{(1)}$$

$$= \left(\frac{\langle 1 | \langle 1 |}{\langle 3 |} + \frac{\langle 3 | \langle 3 |}{\langle 3 |} + \frac{\langle 3 | \langle 1 |}{\langle 3 | \langle 1 |} \right) \langle 1 |$$

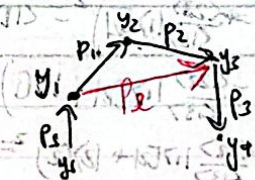
$$= \frac{1 \langle 1 |}{\langle 3 |} - \frac{1 \langle 1 |}{\langle 3 |} - \frac{\langle 3 | \langle 1 |}{\langle 3 | \langle 1 |} \langle 1 |$$

I feel like if I understood the sentence "we can simply pick the momentum twistor of the loop line l to be $\tilde{z}_1 = (1, [1])$ " on page 164, I could answer this ~~whole~~ question

Copy backwards: this new:

$$[1]_1^a = \langle 1 | \tilde{a} y_1^{aa} = \langle 1 | \tilde{a} y_3^{aa}$$

$$\text{or } p_1 = y_1^A - y_3^A$$



at fixed beam $(AB) = (1, 3)$ is a line in momentum twistor space $\Leftrightarrow$ point in dual space

whereas here we have a single monom $\Leftrightarrow$ 2 points in dual space $\Leftrightarrow$ point in momentum twistor space

$$C = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

May the other solution (2)
 $-(|1\rangle + \frac{[23]}{[13]} |2\rangle) |1\rangle = -(1 + \frac{[23]}{[13]} |2\rangle)$

$$[23] = \frac{(\langle 12 | [3] + \langle 31 | [2] + \langle 23 | [1]) (\langle 23 | [1] + \langle 12 | [3] + \langle 31 | [2])}{\langle 12 | [23] + \langle 23 | [12] + \langle 31 | [31]}}$$

$$= \frac{\langle 12 | [23] [3] + \langle 31 | [23] [2] + \langle 23 | [23] [1] + \langle 23 | [12] [3] + \langle 23 | [31] [2] + \langle 23 | [23] [12]}{(\langle 12 | [23] + \langle 23 | [12] + \langle 31 | [31])}$$

$$= \frac{\langle 12 | [23] [3] + \langle 31 | [23] [2] + \langle 23 | [23] [1] + \langle 23 | [12] [3] + \langle 23 | [31] [2] + \langle 23 | [23] [12]}{\langle 12 | [23] + \langle 23 | [12] + \langle 31 | [31]}$$

Or

$$[13] = \frac{(\langle 51 | [2] + \langle 25 | [1] + \langle 12 | [5]) (\langle 23 | [1] + \langle 12 | [3] + \langle 31 | [2])}{\langle 51 | [23] + \langle 25 | [12] + \langle 12 | [31]}$$

$$= \frac{\langle 51 | [23] [2] + \langle 25 | [23] [1] + \langle 12 | [23] [5] + \langle 51 | [12] [3] + \langle 25 | [12] [2] + \langle 12 | [12] [5]}{\langle 51 | [23] + \langle 25 | [12] + \langle 12 | [31]}$$

$$= \frac{\langle 51 | [23] [2] + \langle 25 | [23] [1] + \langle 12 | [23] [5] + \langle 51 | [12] [3] + \langle 25 | [12] [2] + \langle 12 | [12] [5]}{\langle 51 | [23] + \langle 25 | [12] + \langle 12 | [31]}$$

Ex 6.4
w.r. I & p. 100
 $\langle A, B \rangle (123) = 0$

$$\langle A, B \rangle (123) = \langle A, B \rangle (123) + \langle A, B \rangle (132) + \langle A, B \rangle (213) + \langle A, B \rangle (231) + \langle A, B \rangle (312) + \langle A, B \rangle (321)$$

$$(123) \cap (345) = (12)(345) + (31)(2345) + (23)(1345)$$

$$\text{Thus: } \langle A, B \rangle (123) \cap (345) = \langle A, B \rangle (123) + \langle A, B \rangle (132) + \langle A, B \rangle (213) + \langle A, B \rangle (231) + \langle A, B \rangle (312) + \langle A, B \rangle (321)$$

So if $\langle A, B \rangle = \langle B \rangle$: then it zero

$$\text{If } \langle A, B \rangle = \langle B \rangle : \text{ then it zero}$$

Then (123) might be:

$$\langle A, B \rangle (123) \cap (345) = \langle A, B \rangle (123) + \langle A, B \rangle (132) + \langle A, B \rangle (213) + \langle A, B \rangle (231) + \langle A, B \rangle (312) + \langle A, B \rangle (321)$$

$$= (2531)(1234)(2345) + (2823)(1234)(1345) + (1231)(1234)(2345) + (1222)(1234)(1345)$$

So the full thing is:

$$(2531)(1234)(2345)(2451)$$

$$\text{What if we write it as: } (23)(4512) + (42)(13512) + (34)(2512)$$

$\langle A, B \rangle$

prob: $\langle A, B \rangle = 0$ no matter what

$$\text{hence: } \langle A, B \rangle (123) \cap (345) + \langle A, B \rangle (132) \cap (345) + \langle A, B \rangle (213) \cap (345) + \langle A, B \rangle (231) \cap (345) + \langle A, B \rangle (312) \cap (345) + \langle A, B \rangle (321) \cap (345)$$

$$\langle A, B \rangle (123) \cap (345) + \langle A, B \rangle (132) \cap (345) + \langle A, B \rangle (213) \cap (345) + \langle A, B \rangle (231) \cap (345) + \langle A, B \rangle (312) \cap (345) + \langle A, B \rangle (321) \cap (345)$$

$$\rightarrow \frac{(2531)(1234)(2345)(2451)}{\langle A, B \rangle (123) \cap (345)} \text{ blarup}$$

can we take veridus for these 4?

$$\text{Then we just have } \frac{1}{\langle A, B \rangle (123) \cap (345)} (2531)(1234)(2345)(2451)$$

$$= \frac{(2531)(1234)(2345)(2451)}{(5123)(1234)(4512)(5234)} = \frac{(2531)(1234)(2345)(2451)}{[-(2531)(1234)][-(2451)]} = -1$$

$$(8.35) \begin{matrix} 1 & 2 & 3 \\ \diagdown & \diagup & \diagdown \\ 3 & 2 & 1 \end{matrix} =$$

$$= I_{S, \text{top}} (12, 34) + I_{S, \text{right}} + I_{S, \text{bot}} (34, 51)$$

$$= \frac{\langle 5123 \rangle \langle 1234 \rangle}{\langle AB51 \rangle \langle AB12 \rangle \langle AB23 \rangle \langle AB34 \rangle} + \frac{\langle 2345 \rangle \langle 3451 \rangle}{\langle AB23 \rangle \langle AB34 \rangle \langle AB45 \rangle \langle AB51 \rangle} + \frac{\langle AB1 \rangle \langle 123 \rangle \langle 345 \rangle \langle 2451 \rangle}{\langle AB12 \rangle \langle AB23 \rangle \langle AB34 \rangle \langle AB45 \rangle \langle AB51 \rangle}$$

$$\downarrow 1 \rightarrow 14$$

$$\frac{\langle 1234 \rangle \langle 2345 \rangle}{\langle AS12 \rangle \langle AB23 \rangle \langle AB34 \rangle \langle AB45 \rangle} + \frac{\langle 3451 \rangle \langle 4512 \rangle}{\langle AB34 \rangle \langle AB45 \rangle \langle AB51 \rangle \langle AB12 \rangle}$$

denominator clearly invariant under cyclic rotation

numerator or $(34)(5123) + (51)(4123)$

$$= (12)(345) + (31)(2345) + (23)(1345)$$

$$= \frac{[AB31](2345) + [AB23](1345)] \langle 2451 \rangle}{+ [AB31](2345)(2451) + [AB23](1345)(2451)}$$

$$\frac{\langle 5123 \rangle \langle 1234 \rangle \langle AB45 \rangle + \langle 2345 \rangle \langle 3451 \rangle \langle AB12 \rangle}{\langle AB12 \rangle \langle AB23 \rangle \langle AB34 \rangle \langle AB45 \rangle \langle AB51 \rangle}$$

hmm: $\langle AB45 \rangle \langle 1234 \rangle \langle 5123 \rangle + [AB12](2345)(3451) + [AB31](2345)(2451) + [AB23](1345)(2451)$

$$\downarrow 12345 \rightarrow 23451$$

$$[AB51] \langle 2345 \rangle \langle 1234 \rangle + [AB23] \langle 3451 \rangle \langle 4512 \rangle + [AB45] \langle 3451 \rangle \langle 3512 \rangle + [AB34] \langle 2451 \rangle \langle 3512 \rangle$$

$$AB \cdot \text{Plucker} = 0$$

$$\langle 51 \rangle \langle 2345 \rangle$$

$$\langle AB1 \rangle (\langle 12345 \rangle + \langle 51234 \rangle + \langle 45123 \rangle + \langle 34512 \rangle + \langle 24513 \rangle) = 0$$

$$[AB15] \langle 1234 \rangle + [AB14] \langle 5123 \rangle + [AB13] \langle 4512 \rangle + [AB12] \langle 3451 \rangle = 0$$

$$\langle AB45 \rangle$$

$$(2345) [AB15] \langle 1234 \rangle + [AB14] \langle 5123 \rangle + [AB45] \langle 1234 \rangle \langle 5123 \rangle$$

so we now have to show:

$$\langle AB45 \rangle \langle 1234 \rangle \langle 5123 \rangle + \langle AB34 \rangle \langle 1234 \rangle \langle 2345 \rangle = \langle AB45 \rangle \langle 3451 \rangle \langle 3512 \rangle + \langle AB34 \rangle \langle 2451 \rangle \langle 3512 \rangle$$

$$(1234) (\langle AB45 \rangle \langle 5123 \rangle + \langle AB34 \rangle \langle 2345 \rangle)$$

$$(3512) (\langle AB45 \rangle \langle 3451 \rangle + \langle AB34 \rangle \langle 2451 \rangle)$$

$$(AB45) : (AB41) \langle 2345 \rangle + (AB45) \langle 1234 \rangle + (AB43) \langle 4512 \rangle + (AB42) \langle 3451 \rangle$$

$$- (3512) [\langle AB41 \rangle \langle 2345 \rangle + \langle AB45 \rangle \langle 1234 \rangle]$$

$$\text{Need to show: } (AB51) \langle 1234 \rangle \langle 2345 \rangle = (AB41) \langle 2345 \rangle \langle 5123 \rangle$$

$$(AB51) \langle 1234 \rangle = (AB41) \langle 5123 \rangle$$

$$(AB15) \langle 1234 \rangle = (AB14) \langle 5123 \rangle$$

$$\text{Yes! because } \langle 51 \rangle \langle 1234 \rangle - \langle 4 \rangle \langle 5123 \rangle = 0$$

$$\text{and } AB \text{ is not } (13) \text{ or } (512) \cap (234)$$

$$(25) \langle 1234 \rangle + (42) \langle 1234 \rangle : \text{different with } (51) \text{ or } (41) = 0$$

$$\text{problem how } AB = (24) \text{ or } (123) \cap (345) = (31) \langle 2345 \rangle + (23) \langle 1345 \rangle$$

Wait! If $AB = A \circ B$, then we couldn't do the cancel in the first place (and then even the denominator would change) matching with the Plucker relations (which we can prove are really dual via Plucker's theorem) but we can't cancel since $\langle AB12 \rangle$ on the left $\neq \langle AB125 \rangle$ on the right

If this argument works, then it is general, since we can just write $\langle 1, 11, 112, 113, 114 \rangle$ $\langle 11, 112, 113, 114, 1 \rangle$ and do it to general mod 5

problem: AB only has 45
if $AB = (13)$
not the other situation

problem 2: different

the plane does AB1 contain this point?

If $AB = 13$ then clearly $AB1 = 0$

Chapter 9

BA p184: "the product of delta functions is invariant under a GL(k) rotation of the k a-indices."

$\prod_{a=1}^k \delta^a(\sum_{i=1}^n C_{ai} W_i)$ If we just consider the 4 basis capitals $A=1,2,3,4$ of W

Let $k=2, n=5$, so $C = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \end{pmatrix}$ $W = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \end{pmatrix}$

Take $\begin{pmatrix} x & y \\ u & v \end{pmatrix} \in GL(2)$

$\downarrow$
 $\begin{pmatrix} r_{C1} + s_{C21} & r_{C2} + s_{C22} & \dots \\ r_{C4} + s_{C41} & \dots & \dots \end{pmatrix}$

So the first delta function was:

① $\delta(C_{11} C_{11}' + C_{12} C_{21}' + C_{13} C_{31}' + C_{14} C_{41}' + C_{15} C_{51}') \delta(C_{21} C_{11}' + C_{22} C_{21}' + C_{23} C_{31}' + C_{24} C_{41}' + C_{25} C_{51}')$

$\downarrow$
 $\delta((r_{C1} + s_{C21}) C_{11}' + (r_{C2} + s_{C22}) C_{21}' + \dots)$

② $\delta(r(C_{11} C_{11}' + C_{12} C_{21}' + C_{13} C_{31}' + C_{14} C_{41}' + C_{15} C_{51}') + s(C_{21} C_{11}' + C_{22} C_{21}' + C_{23} C_{31}' + C_{24} C_{41}' + C_{25} C_{51}'))$

① $\Rightarrow$ ② obviously

② $\Rightarrow$ ① ?

$av_1 + bv_2 = 0$
 $cv_1 + dv_2 = 0 \Rightarrow -\frac{a}{c}(cv_1 + dv_2) = 0 \Rightarrow -av_1 - \frac{ad}{c}v_2 = 0$
 $(b - \frac{ad}{c})v_2 = 0$

$ad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} r & s \\ u & v \end{pmatrix} \in GL(2)$ so $ad - bc \neq 0$
 $bc - ad \neq 0$
 $b - \frac{ad}{c} \neq 0$

$\Rightarrow$ so $v_2 \neq 0$
 similarly, $v_1 \neq 0$

Problem 9.1 $\prod_{a=1}^k \delta^a(\sum_{i=1}^n C_{ai} W_i) = \sum_{i=1}^n \delta^a(W_i) C_{ai}$ Pair function $\prod_{a=1}^k \delta^a(W_i) = \sum_{a=1}^k \delta^a(W_i)$

$\int d^2 \tilde{u}_j \rangle e^{-i \langle \tilde{u}_j | \tilde{u}_j \rangle}$
 Take $a=1$: $\delta^1(\tilde{u}_1) + C_{11} \tilde{u}_1 + C_{12} \tilde{u}_2 + C_{13} \tilde{u}_3 + C_{14} \tilde{u}_4 + C_{15} \tilde{u}_5$
 $\int d^2 \tilde{u}_j \rangle e^{-i \langle \tilde{u}_1 | \tilde{u}_j \rangle - i \langle \tilde{u}_j | \tilde{u}_j \rangle} \delta^2(\tilde{u}_1) + C_{11} \tilde{u}_1 + C_{12} \tilde{u}_2 + \dots$
 $= \delta^2(\langle \tilde{u}_1 | \tilde{u}_1 \rangle + C_{11} \langle \tilde{u}_1 | \tilde{u}_1 \rangle + \dots)$

$\delta^2(\lambda, \lambda)$

Draw on Grassmanns for Scatly Operators in N=4 p11:

$\delta^{2k}(B_{\alpha\beta} \tilde{u}_i) = \int d^2 \tilde{u}_i \sigma_{\alpha\beta} \delta^{2k}(\tilde{u}_i - \sigma_{\alpha\beta} B_{\alpha\beta}^{-1})$

Work out an example on the next page

$$\delta^{2k}(B\tilde{u}) = \int d^{2n}x \delta_{\tilde{\alpha}} \delta^{2n}(\tilde{u} - \sigma_{\tilde{\alpha}} B_{\tilde{\alpha}i}^{\perp})$$

$$\int f(\tilde{u}_i) \delta^{2k}(B_{\tilde{\alpha}i} \tilde{u}_i) d^{2n} \tilde{u}_i$$

say $k=2$, $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_1^x & \mu_2^x & \mu_3^x & \mu_4^x \\ \mu_1^y & \mu_2^y & \mu_3^y & \mu_4^y \end{pmatrix}$

$$\prod_i f(\mu_1^x, \mu_1^y, \mu_2^x, \mu_2^y, \mu_3^x, \mu_3^y, \mu_4^x, \mu_4^y) = f(0, 0, 0, \mu_1^x, \mu_2^y, \mu_4^x, \mu_4^y) \delta(0)^4?$$

Rght: $\int f(\tilde{u}) \int d^{2n}x \delta_{\tilde{\alpha}} \delta^{2n}(\tilde{u} - \sigma_{\tilde{\alpha}} B_{\tilde{\alpha}i}^{\perp})$, $B^{\perp} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$\delta^{2n}(\tilde{u}_i - \sigma_i B_{i1}^{\perp} - \sigma_2 B_{21}^{\perp})$$

$$\delta(\tilde{u}_1^x - \sigma_1 B_{11}^{\perp} - \sigma_2 B_{21}^{\perp})$$

check: $\tilde{u} = (\tilde{u}_1^x, \mu_1^y)$, but needs to place to x or y on the right side?

* I guess it has to be: $\delta(\tilde{u}_1^x - \sigma_1 B_{11}^{\perp} - \sigma_2 B_{21}^{\perp})$ since there are 2 (x) σ 's

Here: $\delta(\tilde{u}_1^x), \delta(\tilde{u}_1^y), \delta(\tilde{u}_2^x), \delta(\tilde{u}_2^y)$

$$\delta(\tilde{u}_3 - \sigma_1 B_{13}^{\perp} - \sigma_2 B_{23}^{\perp}) = \delta(\tilde{u}_3^x - \sigma_1^x) \delta(\tilde{u}_3^y - \sigma_1^y)$$

$$\delta(\tilde{u}_4 - \sigma_1 B_{14}^{\perp} - \sigma_2 B_{24}^{\perp}) = \delta(\tilde{u}_4^x - \sigma_2^x) \delta(\tilde{u}_4^y - \sigma_2^y)$$

$$\int f(\mu_1^x, \mu_1^y, \mu_2^x, \mu_2^y, \mu_3^x, \mu_3^y, \mu_4^x, \mu_4^y) \int d^{2n}x \delta_{\tilde{\alpha}} \delta(\tilde{u}_1^x) \delta(\tilde{u}_1^y) \delta(\tilde{u}_2^x) \delta(\tilde{u}_2^y) \delta(\tilde{u}_3^x - \sigma_1^x) \delta(\tilde{u}_3^y - \sigma_1^y) \delta(\tilde{u}_4^x - \sigma_2^x) \delta(\tilde{u}_4^y - \sigma_2^y)$$

$$= \int f(\mu_1^x, \mu_1^y, \mu_2^x, \mu_2^y, \mu_3^x, \mu_3^y, \mu_4^x, \mu_4^y)$$

$$\int d\mu_1 d\mu_2 d\mu_3 d\mu_4 \int d\sigma_{\tilde{\alpha}} f(\mu_1^x, \mu_1^y, \mu_2^x, \mu_2^y, \mu_3^x, \mu_3^y, \mu_4^x, \mu_4^y) \delta(\mu_1) \delta(\mu_2) \delta(\mu_3 - \sigma_1) \delta(\mu_4 - \sigma_1) \delta(\mu_3 - \sigma_2) \delta(\mu_4 - \sigma_2) = f(y)$$

$$= \int d\sigma_{\tilde{\alpha}} f(0, 0, 0, \sigma_1^x, \sigma_1^y, \sigma_2^x, \sigma_2^y)$$

$$\int d^2 \tilde{u}_i \otimes \langle \tilde{u}_i \rangle \int d^{2n}x \delta_{\tilde{\alpha}} \delta^{2n}(\tilde{u}_i - \sigma_{\tilde{\alpha}} B_{\tilde{\alpha}i}^{\perp})$$

$$e^{-i \tilde{u}_i \cdot \mu_i} \delta(\tilde{u}_i - \sigma_{\tilde{\alpha}} B_{\tilde{\alpha}i}^{\perp})$$

$$\int d\sigma_{\tilde{\alpha}} \delta(\tilde{u}_i - \sigma_{\tilde{\alpha}} B_{\tilde{\alpha}i}^{\perp})$$

$$\int dy f(y_1, y_2) \int dx \delta(x - y_1) \delta(x - y_2) \delta(0) |_{\sigma = \tilde{u}_i}$$

$$\int d^2k (B_{\tilde{\alpha}i} \tilde{u}_i)$$

$$B_{\tilde{\alpha}1} \tilde{u}_1 + B_{\tilde{\alpha}2} \tilde{u}_2 + \dots + B_{\tilde{\alpha}n} \tilde{u}_n = 0$$

$$\int d\sigma e^{i \sigma \cdot B^{\perp} \cdot \lambda} = \delta(B^{\perp} \cdot \lambda)$$

$$\mu = \sigma B^{\perp}$$

fixed/solved on page 197

$$\mu^{\perp} B$$

$$\begin{pmatrix} \mu^x \\ \mu^y \end{pmatrix} \in B^{\perp} \iff \begin{pmatrix} \mu^x \\ \mu^y \end{pmatrix} \cdot \begin{pmatrix} B^{\perp} \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} \mu^x \\ \mu^y \end{pmatrix} = \sigma_1^x B_1^{\perp} + \sigma_2^x B_2^{\perp} + \dots + \sigma_n^x B_n^{\perp}$$

$$\begin{pmatrix} \mu^y \\ \mu^x \end{pmatrix} = \sigma_1^y B_1^{\perp} + \sigma_2^y B_2^{\perp} + \dots + \sigma_n^y B_n^{\perp}$$

PH would have had 3.3 $\int d^2 p_0 \delta^{2n}(A_j - p_0 B_{0j})$ directly from the inverse Fourier integral of the function space integral $\int_{B=1} d^2 \tilde{\mu}_i e^{i \lambda_i \tilde{\mu}_i} \mathcal{L}_{h,i}(W)$

$$= \left(\prod_{i=1}^n \int d^2 \tilde{\mu}_i e^{i \lambda_i \tilde{\mu}_i} \right) \int \frac{d^{2n} B}{\text{GL}(K)} \frac{\delta^{4E+4F}(B, W)}{m_1 m_2 \dots m_n} \leftarrow \text{by unitarity}$$

$\delta^{2E}(B, \tilde{\mu})$ as $\int d^2 p_\alpha e^{-i p_\alpha B_{0j} \tilde{\mu}_j}$ and then carry out the $2n$ Fermi integrals in 3d

$$\delta^{4E+4F}(B, W) = \delta^{2E}(B, \tilde{\mu}) \delta^{2E}(B, \tilde{\lambda}) \delta^{4E}(B, \tilde{\eta}_i)$$

Recall: $\frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(x-x')} = \delta(x-x')$

$\frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ikf(x',x)} = \delta(f(x',x))$

$\int d^2 p_\alpha e^{-i p_\alpha B_{0j} \tilde{\mu}_j}$

$2E$ delta functions: $\delta(B)$

$$\begin{aligned} & \prod_{i=1}^n \int d^2 \tilde{\mu}_i e^{i \lambda_i \tilde{\mu}_i} \int d^2 p_\alpha e^{-i p_\alpha B_{0j} \tilde{\mu}_j} \\ &= \prod_{j=1}^n \int d^2 p_\alpha \int d^2 \tilde{\mu}_j e^{i (\lambda_j - p_\alpha B_{0j}) \tilde{\mu}_j} \\ &= \int d^2 p_\alpha \delta^2(\lambda_j - p_\alpha B_{0j}) \end{aligned}$$

$$(2.12) \mathcal{L}_{h,i}(W) = \int \frac{d^{2n} B}{\text{GL}(K)} \frac{\delta^{4E+4F}(B, W)}{m_1 m_2 \dots m_n} = \int \frac{d^{2n} B}{\text{GL}(K)} \delta^{2E}(B, \tilde{\mu}) \delta^{2E}(B, \tilde{\lambda}) \delta^{4E}(B, \tilde{\eta}_i)$$

Unit: $\int d^2 \tilde{\mu}_i e^{i \lambda_i \tilde{\mu}_i} \delta^{2n}(\tilde{\mu}_i - \sigma_\alpha B_{0i}^\perp) = e^{i (\tilde{\mu}_i - \sigma_\alpha B_{0i}^\perp) \lambda_i}$

$W_{\alpha i} = \delta^{2n}(\tilde{\mu}_i - \sigma_\alpha B_{0i}^\perp)$

Back to exercise 9.1 $\prod_{a=1}^n \delta^2(|\tilde{\mu}_a\rangle + \sum_{b=1}^n |\mu_b\rangle c_{ab})$

$$\delta^2(\mu, c) = \delta^2(\sum_{i=1}^n \mu_i c_{ia})$$

$$\int d^2 \mu_i e^{i \mu_i c_{ia}} \int d^2 \mu_j e^{i \mu_j c_{ja}} \dots \int d^2 \mu_k e^{i \mu_k c_{ka}}$$

Since $a \in 1, \dots, K$ has been integrated out, we have $\prod_{r=1}^K \int d^2 a_r \delta^2(c_{ra} a_r + i r^2) e^{i \mu^r (c_{ra} a_r - i r^2)}$

$\delta(c_{ra} a_r + i r^2) \delta^2(\mu^r c_{ra} a_r - i r^2)$

$\downarrow$
 $\langle 2058 \rangle \langle 2051 \rangle$
 $\langle 2042 \rangle \langle 2049 \rangle \langle 2039 \rangle$

$$= \begin{pmatrix} 1 & 0 & C_{11} & C_{12} \\ 0 & 1 & C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ Q_1 \\ Q_2 \end{pmatrix}$$

$$\begin{pmatrix} -C_1, H1 & -C_2, H1 & C_3, H1 & 1 \\ -C_1, H2 & -C_2, H2 & C_4, H2 & 0 \\ -C_1, n & -C_2, n & C_5, n & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 12 \\ 13 \end{pmatrix}$$

$$\begin{pmatrix} -C_1, K1 & -C_2, K1 & C_3, K1 & 1 \\ -C_1, K2 & -C_2, K2 & C_3, K2 & 0 \\ -C_1, n & -C_2, n & C_3, n & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$C_0 = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

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TYPE-4

(b) 8 Lax

$$a \int dx \delta(ax) = \int dx \delta(x)$$

so $\delta(ax) = \frac{\delta(x)}{|a|}$ and $\delta(ax) = \delta(-ax)$

If a is negative: dC_{axis}

$$f(x) = f(a) + f'(a) + \frac{f''(a)}{2!} + \frac{f'''(a)}{3!} + \dots$$

$$\int \frac{d}{dx} \delta(x-x') = - \int \delta(x-x') \frac{d}{dx} f(x)$$

$$f(x) = \lim_{n \rightarrow \infty} f_n(x)$$

$$\sum_i \int dx \frac{f(x_i)}{|df/dx|} g(x) = \sum_i \frac{g(x_i)}{\frac{df}{dx}(x_i)}$$

$$\int \frac{d}{dx} \delta(x) dx = \int \frac{d}{dx} \delta(x) dx = \int \frac{d}{dx} \delta(x) dx$$

$$\delta(f(a) + f'(a)x + \frac{f''(a)}{2!}x^2 + \dots)$$

$$= \sum_i \frac{dP}{dx}(x_i) \left| \frac{dP}{dx}(x_i) \right|$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

smallest example:

$$C = \begin{pmatrix} C_{11} & C_{12} & \frac{427B_{13} + 6237B_{14}}{227(347)} & \frac{4487B_{13} + 6537B_{14} + 347B_{15}}{247(247)} & \frac{6517B_{14} + 4147B_{15}}{447(357)} \\ & & \frac{4427B_{23} + 6237B_{24}}{227(347)} & \frac{4457B_{23} + 6537B_{24} + 347B_{25}}{247(247)} & \frac{6517B_{24} + 4147B_{25}}{447(357)} \end{pmatrix}$$

$$\text{we: } \delta(f(x)) = \sum_i \frac{\delta(x-x_i)}{|\frac{df}{dx}(x_i)|}$$

$$\int \delta(f(x)) \delta(f(x)) = 1$$

$$\int \frac{df}{dx}(x) dx \delta(f(x))$$

$$\text{near } x_1: f(x) \approx f(x_1) + f'(x_1)(x-x_1) + \frac{f''(x_1)}{2!}(x-x_1)^2 + \frac{f'''(x_1)}{3!}(x-x_1)^3 + \dots$$

$$\int \sum_i \frac{\delta(x-x_i)}{|\frac{df}{dx}(x_i)|} dx = \sum_i \frac{1}{|\frac{df}{dx}(x_i)|}$$

$$\int \sum_i \frac{\delta(x-x_i)}{|\frac{df}{dx}(x_i)|} g(x) dx = \sum_i \frac{g(x_i)}{|\frac{df}{dx}(x_i)|}$$

$$\int dx \delta(f(x)) g(x) = \int -\frac{df}{f'(y)} \delta(y) g(f(y))$$

$$y=f(x)$$

$$x=f(y)$$

$$dx = \frac{df}{f'(y)} = \frac{1}{f'(y)} \frac{df}{dy} dy = \frac{1}{f'(y)} \frac{df}{dy} dy$$

$$= \int \frac{df}{dy} dy \frac{1}{f'(y)} g(f(y)) \delta(y)$$

$$= -\frac{df}{dy} \frac{1}{f'(y)} g(f(y))$$

$$\int \frac{df}{dx}(x) dx \delta(f(x) + f'(x)(x-x_1) + \frac{f''(x_1)}{2!}(x-x_1)^2)$$

$$\approx \int \frac{df}{dx}(x) dx \delta(f'(x_1)(x-x_1))$$

$$= \int \frac{df}{dx}(x) dx \delta(x-x_1) \Rightarrow \delta(f(x)) = \frac{\delta(x-x_1)}{|\frac{df}{dx}(x_1)|} \text{ when } x \rightarrow x_1$$

the not be a spike near x_1 for each zero

$$\delta(x-x_1) = \frac{d}{dx} \theta(x-x_1) \quad \theta(x-x_1) = \begin{cases} 0 & x < x_1 \\ 1 & x > x_1 \end{cases}$$

$$\int_{-\infty}^{\infty} \frac{d}{dx} \theta(x-x_1) f(x) dx = \int_{-\infty}^{\infty} \theta(x-x_1) f'(x) dx = \int_{x_1}^{\infty} f'(x) dx = f(x) \Big|_{x_1}^{\infty} = f(\infty) - f(x_1)$$

Ex 9.3

$$(9.30) \delta^2(117-117C_{11}-127C_{21}) = \frac{1}{427} \delta(117-\frac{417}{227}) \delta(127-\frac{417}{227})$$

$$417-617C_{11}-427C_{21}$$

$$117^9-117^9C_{11}-127^9C_{21}$$

$$117^{10}-117^{10}C_{11}-127^{10}C_{21}$$

$$411 \cdot 117 = 411^{11} 117 - 411^{11} 127$$

$$\delta(f(g)) = \frac{\delta(g)}{\frac{df}{dg}}$$

$$417-617C_{11}-427C_{21} \rightarrow 417-427C_{21} = 417 \left(\frac{417-627}{427} \right)$$

$$\delta\left(417 \left(\frac{417-627}{427} \right)\right) = \frac{1}{|417|} \delta\left(\frac{417}{427} - C_{21}\right) \text{ by uniq rule}$$

$$\text{which } \frac{1}{427^2}$$

$$\text{mass: } J = \left| \frac{\partial^2 f}{\partial x \partial g} \frac{dy}{dg} \right|$$

$$\begin{aligned}
 (9.31) \quad & \delta^2 \left(|1\rangle + \sum_{l=3}^n c_{1l} |l\rangle \right) \delta^2 \left(|2\rangle + \sum_{l=3}^n c_{2l} |l\rangle \right) \\
 &= \delta^2 \left(|1\rangle + \sum_{l=3}^n \frac{\langle l2 \rangle}{\langle 22 \rangle} |l\rangle \right) \delta^2 \left(|2\rangle + \sum_{l=3}^n \frac{\langle l1 \rangle}{\langle 21 \rangle} |l\rangle \right) \\
 &= \delta^2 \left(\frac{1}{\langle 12 \rangle} \left(|1\rangle \langle 12 \rangle + \sum_{l=3}^n |l\rangle \langle l2 \rangle \right) \right) \delta^2 \left(\frac{1}{\langle 21 \rangle} \left(|2\rangle \langle 21 \rangle + \sum_{l=3}^n |l\rangle \langle l1 \rangle \right) \right) \\
 &= |\langle 12 \rangle|^2 |\langle 21 \rangle|^2 \delta^2 \left((-1) \left(|2\rangle + \sum_{l=3}^n \frac{\langle l2 \rangle}{\langle 22 \rangle} |l\rangle \right) \right) \delta^2 \left((-1) \left(|1\rangle + \sum_{l=3}^n \frac{\langle l1 \rangle}{\langle 21 \rangle} |l\rangle \right) \right)
 \end{aligned}$$

$\uparrow$
 ϕ -comp and this
 with this, becomes a 2-comp and this

$$\begin{aligned}
 (9.32) \quad & \prod_{a=1}^2 \delta^{(4)} \left(n_a + \sum_{l=3}^n c_{al} n_l \right) \\
 &= \delta^{(4)} \left(n_1 + \sum_{l=3}^n c_{1l} n_l \right) \delta^{(4)} \left(n_2 + \sum_{l=3}^n c_{2l} n_l \right) \\
 &= \delta^{(4)} \left(n_1 + \sum_{l=3}^n \frac{\langle l2 \rangle}{\langle 22 \rangle} n_l \right) \delta^{(4)} \left(n_2 + \sum_{l=3}^n \frac{\langle l1 \rangle}{\langle 21 \rangle} n_l \right) \\
 &= \delta^{(4)} \left(\frac{\langle 22 \rangle}{\langle 22 \rangle} \left(\langle 12 \rangle n_1 + \sum_{l=3}^n \langle l2 \rangle n_l \right) \right) \delta^{(4)} \left(\frac{\langle 21 \rangle}{\langle 21 \rangle} \left(\langle 21 \rangle n_2 + \sum_{l=3}^n \langle l1 \rangle n_l \right) \right) \\
 &= |\langle 12 \rangle|^4 |\langle 21 \rangle|^4 \delta^{(4)} \left((-1) \langle 21 \rangle \cdot (n_1 |1\rangle + \dots + n_n |n\rangle) \right) \delta^{(4)} \left((-1) \langle 11 \rangle \cdot (n_2 |2\rangle + \dots + n_n |n\rangle) \right)
 \end{aligned}$$

$$|1\rangle |2\rangle \dots |n\rangle$$

$$M_1 = |1\rangle |2\rangle^2 - |2\rangle |1\rangle^2$$

$$|1\rangle |2\rangle + |2\rangle |1\rangle + |2\rangle |3\rangle = 0$$

$$|1\rangle |2\rangle + |2\rangle |3\rangle = -|3\rangle |2\rangle$$

$$|1\rangle \frac{\langle 22 \rangle}{\langle 22 \rangle} + |2\rangle \frac{\langle 32 \rangle}{\langle 22 \rangle} = -|3\rangle$$

could just multiply eqn by $\frac{1}{\sqrt{\langle 22 \rangle}}$

$$M_3 = -\frac{\langle 34 \rangle}{\langle 22 \rangle}$$

$$\langle 22 \rangle M_3 = -\langle 34 \rangle$$

$$\text{If } M_3 = \tilde{M}_3$$

$$\langle 12 \rangle \langle 34 \rangle + \langle 14 \rangle \langle 23 \rangle + \langle 13 \rangle \langle 42 \rangle = 0$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} |1\rangle & |2\rangle \\ |1\rangle & |2\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$a|1\rangle + b|1\rangle^2 = 1$$

$$-\frac{1}{\langle 22 \rangle} (a|2\rangle + b|2\rangle^2) = 0$$

$$-a|1\rangle - b \frac{|2\rangle^2 |1\rangle}{|2\rangle^2} = 0$$

$$b(1 - \frac{|2\rangle^2 |1\rangle}{|2\rangle^2}) = 1$$

$$b \left(\frac{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}{|2\rangle^2} \right) = 1$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$b = \frac{|2\rangle^2}{|1\rangle^2 |2\rangle - |2\rangle^2 |1\rangle}$$

$$\begin{pmatrix} c & d \\ a & b \end{pmatrix} \begin{pmatrix} |1\rangle & |2\rangle \\ |1\rangle & |2\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$c|1\rangle + d|1\rangle^2 = 1$$

$$-c|2\rangle - d \frac{|1\rangle^2 |2\rangle}{|1\rangle^2} = 0$$

$$d(1 - \frac{|1\rangle^2 |2\rangle}{|1\rangle^2}) = 0$$

$$d = \frac{|1\rangle^2}{|1\rangle^2 |2\rangle - |1\rangle^2 |2\rangle} = \frac{|1\rangle^2}{\langle 22 \rangle}$$

$$a = -\frac{b|2\rangle^2}{|2\rangle^2}$$

$$a = -\frac{b|2\rangle^2}{|2\rangle^2}$$

$$a = -\frac{b|2\rangle^2}{|2\rangle^2}$$

$$a = -\frac{b|2\rangle^2}{|2\rangle^2}$$

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$$a = -\frac{b|2\rangle^2}{|2\rangle^2}$$

$$a = -\frac{b|2\rangle^2}{|2\rangle^2}$$

$$\frac{1}{\sqrt{12}} \begin{pmatrix} +12^2 & +12^1 \\ +11^2 & -11^1 \end{pmatrix} \begin{pmatrix} 11^1 & 12^1 \\ 11^2 & 12^2 \end{pmatrix}$$

$$= \frac{1}{\sqrt{12}} \begin{pmatrix} 11^1 12^2 + 11^2 12^1 & 12^1 12^2 + 12^2 12^1 \\ -11^1 11^2 + 11^2 11^1 & -11^2 12^1 + 11^1 12^2 \end{pmatrix} = \frac{1}{\sqrt{12}} \begin{pmatrix} 417 & 0 \\ 0 & 417 \end{pmatrix}$$

$$\frac{1}{\sqrt{12}} \begin{pmatrix} -12^2 & +12^1 \\ +11^2 & -11^1 \end{pmatrix} \begin{pmatrix} 13^1 & 14^1 \\ 13^2 & 14^2 \end{pmatrix}$$

$$= \frac{1}{\sqrt{12}} \begin{pmatrix} -13^1 12^2 + 12^1 13^2 & -12^2 14^1 + 12^1 14^2 \\ +11^2 13^1 + 11^1 13^2 & +11^2 14^1 + 11^1 14^2 \end{pmatrix}$$

$$= \frac{1}{\sqrt{12}} \begin{pmatrix} -\langle 32 \rangle & -\langle 42 \rangle \\ +\langle 13 \rangle & -\langle 14 \rangle \end{pmatrix}$$

$$\det \begin{pmatrix} -\frac{\langle 32 \rangle}{\sqrt{12}} & -\frac{\langle 42 \rangle}{\sqrt{12}} \\ -\frac{\langle 13 \rangle}{\sqrt{12}} & -\frac{\langle 14 \rangle}{\sqrt{12}} \end{pmatrix} = \frac{\langle 32 \rangle \langle 14 \rangle}{12} - \frac{\langle 42 \rangle \langle 13 \rangle}{12}$$

$$= \frac{1}{\sqrt{12}} (\langle 32 \rangle \langle 14 \rangle - \langle 42 \rangle \langle 13 \rangle)$$

$$= \frac{1}{\sqrt{12}} (\langle 14 \rangle \langle 32 \rangle - \langle 13 \rangle \langle 42 \rangle)$$

$$= \frac{1}{\sqrt{12}} (\langle 14 \rangle \langle 32 \rangle + \langle 13 \rangle \langle 24 \rangle)$$

$$= \frac{1}{\sqrt{12}} (-\langle 12 \rangle \langle 43 \rangle)$$

$$\frac{1}{\sqrt{12}} \begin{pmatrix} \langle 23 \rangle & \langle 24 \rangle \\ \langle 31 \rangle & \langle 41 \rangle \end{pmatrix} \xrightarrow{\det} \frac{1}{\sqrt{12}} (\langle 23 \rangle \langle 41 \rangle - \langle 24 \rangle \langle 31 \rangle)$$

$$= \frac{1}{\sqrt{12}} (\langle 23 \rangle \langle 41 \rangle + \langle 24 \rangle \langle 13 \rangle)$$

$$= \frac{1}{\sqrt{12}} (-\langle 21 \rangle \langle 34 \rangle)$$

$$= -\frac{\langle 34 \rangle}{\sqrt{12}}$$

Is $(11^1 12^2 - 11^2 12^1)$

$\langle 12 \rangle$ or $\langle 21 \rangle$?

$$\langle 12 \rangle = -\langle 21 \rangle$$

By def

$$\langle 1, 2 \rangle = \langle 1, 1 \rangle + 1 = -\epsilon_{ab} 1^a 1^b + 1$$

$$\langle 1, 1 \rangle = \epsilon_{ab} 1^a 1^b$$

$$\text{ad } \epsilon_{ab} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \epsilon_{12} = -1, \epsilon_{21} = 1$$

$$(9.34) \prod_{i=1}^n M_i = \frac{1}{\sqrt{12}} \begin{pmatrix} \langle 23 \rangle \\ -\frac{\langle 34 \rangle}{\sqrt{12}} \end{pmatrix} \begin{pmatrix} -\frac{\langle 41 \rangle}{\sqrt{12}} \\ -\frac{\langle 12 \rangle}{\sqrt{12}} \end{pmatrix} \dots \begin{pmatrix} -\frac{\langle 12 \rangle}{\sqrt{12}} \end{pmatrix}$$

$$= \frac{(-1)^n}{\sqrt{12}^n} \prod_{i=1}^n \langle 1, i+1 \rangle (\text{mod } n)$$

$$= \frac{(-1)^n}{\sqrt{12}^n} \prod_{i=1}^n \langle 1, i+1 \rangle (\text{mod } n)$$

Plug into 9.29:

$$\int \frac{d^{(n-2)/2} c}{M_1 \dots M_n} \left[\dots \right] = \int \frac{d^{(n-2)/2} c}{M_1 \dots M_n} \frac{1}{\sqrt{12}} \delta(c_1 - \frac{\langle 42 \rangle}{\sqrt{12}}) \delta(c_2 - \frac{\langle 11 \rangle}{\sqrt{12}}) \dots$$

$$= \frac{1}{M_1 \dots M_n} \frac{1}{\sqrt{12}^{n-2}} \delta^4(p) \frac{1}{\sqrt{12}} \delta^4(q) = \frac{\delta^4(p) \delta^4(q)}{\sqrt{12}^n} \cdot \frac{\langle 12 \rangle^n}{(-1)^n} \prod_{i=1}^n c_{i,i+1}$$

$$= \frac{\delta^4(p) \delta^4(q)}{\prod_{i=1}^n \langle 1, i+1 \rangle}$$

Back to det $\mathcal{A}$ - eq. (9.30)

$$\delta^2 \left(\langle 1| - \langle 1| \rangle C_1 - \langle 2| \rangle C_2 \right) \stackrel{?}{=} \frac{1}{\langle 12 \rangle} \delta \left(C_1 - \frac{\langle 12 \rangle}{\langle 1|} \right) \delta \left(C_2 - \frac{\langle 12 \rangle}{\langle 2|} \right)$$

Start exchge: can I say: $\vec{u} = \left(C_1 - \frac{\langle 12 \rangle}{\langle 1|}, C_2 - \frac{\langle 12 \rangle}{\langle 2|} \right)$

$$\begin{pmatrix} \langle 1| & \langle 1| \rangle \\ \langle 2| & \langle 2| \rangle \end{pmatrix} \begin{pmatrix} \langle 1| \rangle C_1 - \langle 12 \rangle C_2 \\ \langle 1| \rangle C_1 - \langle 12 \rangle C_2 \end{pmatrix} = \begin{pmatrix} \langle 1| \rangle - \langle 12 \rangle C_1 - \langle 12 \rangle C_2 \\ \langle 2| \rangle - \langle 21 \rangle C_1 - \langle 21 \rangle C_2 \end{pmatrix} = \begin{pmatrix} \langle 1| \rangle - \langle 12 \rangle C_1 \\ \langle 2| \rangle - \langle 21 \rangle C_1 \end{pmatrix}$$

$$\downarrow \quad \uparrow \frac{1}{\langle 12 \rangle}$$

$$\det = \langle 21 \rangle$$

$$\vec{x} = \vec{u} = \vec{F}(\vec{x})$$

$\vec{F}$ is already linear

$$\text{so we need } \det(\vec{F}) = \frac{1}{\langle 12 \rangle}$$

(9.31)

$$\delta^2 \left(\langle 1| + \sum_{i=3}^n C_i \langle i| \right)$$

$$\downarrow C_i = \frac{\langle 12 \rangle}{\langle i2 \rangle}$$

$$\delta^2 \left(\langle 1| + \sum_{i=3}^n \frac{\langle 12 \rangle}{\langle i2 \rangle} \langle i| \right)$$

$$= \langle 12 \rangle^2 \delta^2 \left(\langle 12 \rangle \langle 1| + \sum_{i=3}^n \langle i2 \rangle \langle i| \right)$$

$$= \langle 12 \rangle^2 \delta^2 \left(\langle 21 \rangle \langle 1| + \sum_{i=3}^n \langle 2i \rangle \langle i| \right)$$

$$\delta^2 \left(\begin{pmatrix} \langle 21 \rangle & \langle 21 \rangle \langle 1| \\ \sum_{i=3}^n \langle 2i \rangle & \sum_{i=3}^n \langle 2i \rangle \langle i| \end{pmatrix} \right) \delta^2 \left(\langle 11 \rangle \langle 1| - \sum_{i=3}^n \langle 1i \rangle \langle i| \right)$$

$$= \delta^2 \left(\begin{pmatrix} \langle 11 \rangle & \langle 11 \rangle \langle 1| \\ \langle 21 \rangle & \langle 21 \rangle \langle 1| \end{pmatrix} \right) \left(\sum_{i=3}^n \langle 1i \rangle \langle i| \right)$$

$$\text{rewrite} \begin{pmatrix} \langle 11 \rangle & 0 & 0 \\ 0 & 0 & \langle 12 \rangle \\ \langle 21 \rangle & \langle 21 \rangle \langle 1| & \langle 21 \rangle \langle 2| \end{pmatrix} \begin{pmatrix} \sum_{i=3}^n \langle 1i \rangle \langle i| \\ \sum_{i=3}^n \langle 1i \rangle \langle i| \\ \sum_{i=3}^n \langle 2i \rangle \langle i| \end{pmatrix} = \begin{pmatrix} \sum_{i=3}^n \langle 1i \rangle \langle i| \\ \sum_{i=3}^n \langle 1i \rangle \langle i| \\ \sum_{i=3}^n \langle 2i \rangle \langle i| \end{pmatrix} = \delta^2 \left(\sum_{i=3}^n \langle 1i \rangle \langle i| \right)$$

$$\det = \langle 11 \rangle (\langle 21 \rangle \langle 12 \rangle - \langle 12 \rangle \langle 21 \rangle)$$

$$+ \langle 12 \rangle (\langle 21 \rangle \langle 12 \rangle - \langle 12 \rangle \langle 21 \rangle)$$

$$+ \langle 11 \rangle \langle 21 \rangle \langle 11 \rangle \langle 21 \rangle + \langle 11 \rangle \langle 21 \rangle \langle 11 \rangle \langle 21 \rangle$$

$$= -(\langle 11 \rangle \langle 21 \rangle)^2 - (\langle 11 \rangle \langle 21 \rangle)^2$$

$$= -2 \langle 11 \rangle \langle 21 \rangle \langle 11 \rangle \langle 21 \rangle$$

$$= -(\langle 11 \rangle \langle 21 \rangle - \langle 11 \rangle \langle 21 \rangle)^2$$

$$= \langle 12 \rangle^2$$

pl90 BH

$$\delta^2 \left(|i\rangle + \sum_j C_{ij} |j\rangle \right)$$

$$\delta^2 \left(|i\rangle + C_{1i} |1\rangle + C_{2i} |2\rangle + C_{3i} |3\rangle + C_{4i} |4\rangle + C_{5i} |5\rangle + C_{6i} |6\rangle \right)$$

$$\delta^2(|i\rangle) = \sum_j |\langle i|j\rangle|^2 C_{ij}^2$$

$$\delta^2(|i\rangle) = C_{2i}^2 |2\rangle^2 + C_{4i}^2 |4\rangle^2 + C_{6i}^2 |6\rangle^2$$

How were these obtained?

$$(9.21) \prod_{a=1}^K \delta^2 \left(\sum_i C_{ai} |i\rangle \right)$$

Here: $K=3$, we have: $\delta^2 \left(\sum_i C_{1i} |i\rangle \right) \delta^2 \left(\sum_i C_{2i} |i\rangle \right) \delta^2 \left(\sum_i C_{3i} |i\rangle \right)$

$$= \delta^2(C_{21} |1\rangle + |2\rangle + C_{23} |3\rangle + C_{25} |5\rangle) \delta^2(C_{41} |1\rangle + C_{43} |3\rangle + |4\rangle + C_{45} |5\rangle)$$

$$\delta^2(C_{61} |1\rangle + C_{63} |3\rangle + C_{65} |5\rangle + |6\rangle)$$

$$= \delta^2(|2\rangle + C_{23} |3\rangle) \delta^2(|4\rangle + C_{43} |3\rangle) \delta^2(|6\rangle + C_{63} |3\rangle)$$

~~$$= \delta^2(C_{21} |1\rangle + |2\rangle + C_{23} |3\rangle + C_{25} |5\rangle) \delta^2(C_{41} |1\rangle + C_{43} |3\rangle + |4\rangle + C_{45} |5\rangle) \delta^2(C_{61} |1\rangle + C_{63} |3\rangle + C_{65} |5\rangle + |6\rangle)$$~~

$$\prod_{i=1,2,3,4,5,6} \delta^2(|i\rangle + C_{ij} |j\rangle)$$

~~$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$~~

~~If the origin for τ is such that~~

~~$$\prod_{a=1}^K \delta^2 \left(\sum_i \tilde{C}_{ai} |i\rangle \right)$$~~

$$C = \begin{pmatrix} C_{21} & 1 & C_{23} & 0 & C_{25} & 0 \\ C_{41} & 0 & C_{43} & 1 & C_{45} & 0 \\ C_{61} & 0 & C_{63} & 0 & C_{65} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -C_{21} & 0 & -C_{23} & 0 & -C_{25} \\ 0 & -C_{41} & 0 & -C_{43} & 0 & -C_{45} \\ 0 & -C_{61} & 0 & -C_{63} & 0 & -C_{65} \end{pmatrix}^T = \tilde{C}$$

so $a=4,5,6$

$$\delta^2(\tilde{C}_{41} |1\rangle + \tilde{C}_{42} |2\rangle + \tilde{C}_{43} |3\rangle + \tilde{C}_{44} |4\rangle + \tilde{C}_{45} |5\rangle + \tilde{C}_{46} |6\rangle)$$

$$= \delta^2(|1\rangle - C_{21} |2\rangle - C_{41} |4\rangle - C_{61} |6\rangle)$$

$$\Rightarrow \delta^2(C_{23} |3\rangle - C_{25} |5\rangle - C_{43} |4\rangle - C_{63} |6\rangle)$$

$$\delta^2(C_{45} |5\rangle - C_{65} |6\rangle - C_{45} |5\rangle - C_{65} |6\rangle)$$

$$\delta^2 \left(|j\rangle - \sum_i C_{ij} |i\rangle \right)$$

Since $C_{25}=0$: $\langle 5| - C_{45} |4\rangle - C_{65} |6\rangle = 0$ & $C_{21} |1\rangle + |2\rangle + C_{23} |3\rangle = 0$

$$\begin{cases} C_{45} \langle 4|5\rangle = -C_{65} \langle 6|5\rangle \\ C_{45} = -C_{65} \frac{\langle 6|5\rangle}{\langle 4|5\rangle} \end{cases}$$

$$C_{21} |1\rangle = -C_{23} |3\rangle$$

$$C_{21} = -\frac{\langle 3|2\rangle}{\langle 1|2\rangle} C_{23}$$

$$\langle 5|4\rangle = \langle 6|4\rangle$$

$$C_{65} = \frac{\langle 5|4\rangle}{\langle 6|4\rangle}$$

(9.23)-(9.24): $\langle 4| + C_{41} |1\rangle + C_{43} |3\rangle + C_{45} |5\rangle = 0$

$\langle 4| + C_{41} |1\rangle + C_{43} |3\rangle + \frac{\langle 5|4\rangle}{\langle 4|5\rangle} |5\rangle = 0$

$$\Rightarrow \langle 4| + C_{43} |3\rangle + \frac{\langle 5|4\rangle}{\langle 4|5\rangle} |5\rangle \Rightarrow C_{43} = -\frac{\langle 5|4\rangle \langle 4|5\rangle - \langle 4|1\rangle}{\langle 4|3\rangle}$$

$$\Rightarrow \langle 4|3\rangle + C_{41} |1\rangle + \frac{\langle 5|4\rangle}{\langle 4|5\rangle} |5\rangle \Rightarrow$$

$$C_{41} = \frac{-\langle 5|4\rangle \langle 5|3\rangle - \langle 4|3\rangle \langle 4|5\rangle}{\langle 4|3\rangle \langle 4|5\rangle}$$

$$= \frac{\langle 6|1\rangle \langle 4|3\rangle}{\langle 4|1\rangle \langle 4|3\rangle}$$

$$\begin{aligned} &= \frac{\langle 5|4\rangle \langle 4|5\rangle - \langle 4|1\rangle \langle 4|3\rangle}{\langle 4|3\rangle \langle 4|5\rangle} \\ &= \frac{\langle 6|1\rangle \langle 4|3\rangle}{\langle 4|1\rangle \langle 4|3\rangle} \end{aligned}$$

BA eq (9.53)

$$C = \begin{pmatrix} [23] & [32] & [12] \end{pmatrix}$$

$C = (C_{23} \ C_{31} \ C_{12})$
 $\rightarrow$ this already takes care of $\int^{2\pi}_0 (G_i G_i)$

then $\int_{\mathbb{R}^2} (\tilde{c}, \langle 1 |)$

$\hat{C}$ is a 2×3 matrix

$$\vec{C} = (\cancel{1} \ \cancel{2} \ \cancel{3}) \ (\cancel{1} \ 2 \ 3)$$

so $\delta^{2,2}(\tilde{c}_i \in 1)$ say: $\mathbb{I}[1+2][2+3][3] = 0$ which is just momentum conservation

Ex 9.6: $\tilde{C}$ is a 2×3 matrix, with the two row vectors spanning a plane that is orthogonal to $C = \begin{pmatrix} [23] & [31] & [12] \end{pmatrix}$

Now, we already know from $\int^{2 \times 1} (C, C1)$, that $C = C^e = \begin{pmatrix} C^1 & C^2 & C^3 \\ C^4 & C^5 & C^6 \end{pmatrix}$ is a 2×3 matrix and two vectors in $\mathbb{R}^3$ that span a plane that is orthogonal to $C = (C^1, C^2, C^3)$.

Therefore, ~~the~~ $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ is a matrix that gives a representation of $\mathbb{Z}_{21}$.

If we plug into (9.52), we find that this term is just monomorphism case ratios

Learn how to deal with Jacobson tactics later

BA 9.7

What does the bosonic delta function for $\langle i |$ look like?

MHV: $\delta^{202} (C|I)$

$\tilde{C}$ is orthogonal:

$\tilde{C} = (\tilde{C}_1, \tilde{C}_2, \tilde{C}_3)$

$\delta^{202} (\tilde{C}_i | I) : \tilde{C} = (\langle 23 \rangle, \langle 13 \rangle, \langle 12 \rangle)$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha_c \end{pmatrix}$

orthogonal
condition

$\begin{pmatrix} 1 & 0 & 0 \\ -\alpha_b & -\alpha_c & 1 \end{pmatrix} = \tilde{C}$ in the MHV case

$\delta^{201} (\tilde{C}_i | I) :$

$\delta^2 (-\alpha_b \langle b | - \alpha_c \langle b | + \langle a |)$

Should, analogous to (9.55) in the MHV case
not be momentum conservation:

$-\alpha_b = \langle ac \rangle$
 $-\alpha_c = \langle ab \rangle$ (or $\langle ca \rangle$ up to sign)

and $1 = \langle bc \rangle$

so we have gauge-fixed $\langle bc \rangle = 1$

$C = \begin{pmatrix} b & c & a \\ 1 & 0 & \alpha_b \\ 0 & 1 & \alpha_c \end{pmatrix}$

$\delta^{202} |I\rangle : \delta^2 : 1 \cdot \bar{b} + 0 \cdot \bar{c} + \alpha_b \cdot \bar{a} = 0$

$\delta^2 : 0 \cdot \bar{b} + 1 \cdot \bar{c} + \alpha_c \cdot \bar{a} = 0$

~~the~~

$\begin{pmatrix} -\beta_b & 1 & 0 \\ -\beta_c & 0 & 1 \end{pmatrix} = \tilde{C}$ in the $\overline{\text{MHV}}$ case

↓

$\delta^2 (-\beta_b \langle a | + \langle b |) \delta^2 (-\beta_c \langle a | + \langle c |)$